Afterword to *Engaging Young Children in Mathematics*  
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**Introduction**

I was initially apprehensive about the invitation to speak at the Conference on Early Childhood Mathematics because I had very little experience working with young children on mathematics. I was invited to the conference as one of the mathematicians, and the other mathematicians expressed similar concerns. Nonetheless, the early childhood teachers, representatives from state education departments, conference organizers and others at the conference consistently reiterated how important it was that mathematicians be involved in efforts to improve early childhood mathematics education.

The conference came at a time when I was about to start working on a grant that would put me on-site at an urban K-8 school – the first time I’d spent significant time at an elementary school since I was a child. I had already been teaching for several years at Wheelock College, a school whose mission is “to improve the lives of children and families,” where the majority of my students are prospective elementary and early childhood teachers. Before the conference, I had been involved several collaborations working to improve the preparation of pre-service and in-service teachers in mathematics and science, but I hadn’t done much work with children.

It’s now been over three years since the conference. Immediately afterward, I started sharing anecdotes and bits of information I learned there with my undergraduate students; the more I know about young children, the more it enhances my credibility with them. On the grant, I’ve been working with graduate students who are doing a full-year internship at the school – helping the interns plan lessons, working with them on their own mathematics understanding, and observing their classes. In talking to them about pedagogy and curriculum for young children, I often find myself referencing things I learned from the conference and this book.

One thing that is clear from both the conference and this book: the project to significantly improve the quality of mathematics education for young children is complex. It can be approached on scales ranging from the individual child, teacher, or parent to the classroom, the school, the community, the state, the nation, and everything in between. It encompasses mathematics, education research, expert practice, knowledge of children and communities, and quite a bit of politics. It is, frankly, too big for any one person to have the expertise to fully grasp; yet it is also a project full of concrete tasks for individuals and teams, with plenty of work for bridge-builders.

This Afterword is one person’s take on the conference and book. I begin with a question to frame the discussion: “Why study mathematics?” and then discuss each of the five sections of this book: Standards in Early Childhood Education; Mathematics Standards; Curriculum, Learning, and Teaching; Professional Development; and Implementation. I focus on significant themes, tensions, things that seem missing, and places where I have something relevant to add.
Why Study Mathematics?

Why is it important to study mathematics? Assumption 2 of this book states that “Pre-kindergarten children have the interest and ability to engage in significant mathematical thinking and learning,” (page 11) beneath which is the additional assumption, perhaps too obvious to state in a book such as this one, that learning mathematics is important.

I offer three answers to the question, “Why study mathematics?” The first is intrinsic interest: mathematics is beautiful, creative, and challenging; in various forms, it has engaged a wide variety of human beings in different cultures and eras. Of course, for too many people, mathematics in school has seemed anything but beautiful and creative. Teachers can best nurture children’s interest in mathematics when they’ve had opportunities to experience it as interesting themselves. In this era of accountability and high stakes testing, it seems especially important to fight for the rights of all people to an intellectual life, to have space to experience and share the awe and wonder that mathematics and other subjects can provide.

The second reason for studying mathematics is because it develops thinking skills. The NCTM Principles and Standards for School Mathematics (PSSM)² process standards—Problem Solving, Communication, Reasoning and Proof, Mathematical Connections, and Representation— all support learning ways of thinking that are clearly relevant beyond mathematics. Process skills are emphasized throughout this book. As Rachelle Feiler reports (page 394), many early childhood educators value the development of critical thinking skills as a goal for their programs, even if they don’t apply this goal in teaching mathematics. I agree with her that “…teachers’ engagement with the process standards can be used as a vehicle to build interest in and develop their understanding of mathematical content” (page 399). As Karen Fuson reports (page 120), even students in traditional classrooms abstract patterns from rote (and often incorrectly) learned methods.

The third reason for studying mathematics is to help us understand and participate in the world in which we live. This justification is the one that appears most frequently in this book, and with good reason, because it is the main entry into mathematics for most children, as they explore and make sense of their world. Understanding mathematics helps us in everyday life, and it can help us describe scientific phenomena, become better citizens, build things, and for some, earn our livings.

Standards in Early Childhood Education

Right from the start, this book makes a distinction between “standards as a vision of excellence and standards as narrow and rigid requirements for mastery,” (page 9) and comes out squarely on the side of standards as a vision of excellence. At the conference itself there was considerable discussion about whether there should be standards at all³, with much of the disagreement stemming from these different usages of the word standards, with all participants concerned that standards not be used to ask children “to perform without understanding” (Richardson page 321).
Sometimes the theme of the conference seemed to be, “Standards are going to happen anyway, so we should be the ones to write them. And here’s why they are a good thing.” As the authors state (pages 9-10), "Admittedly, pressure to create standards also comes from the concern that if experts do not do so, someone with far less experience will,” but they continue, “…We are not merely trying to forestall the disaster of inappropriate standards; we are convinced that articulating standards is a useful and important act.”

The call for standards clearly exists in a political context. The conference focused on mathematics standards and not say, ways of improving the pay, working conditions, and status of early childhood teachers, a more radical (and expensive) move that would clearly do more to improve mathematics education for young children than standards ever could. The necessity of improving conditions for teachers was discussed at the conference; however, absent the resources to address the problem in a meaningful way, we focused primarily on standards -- the topic that the government agency and corporate foundation had sponsored us to address. As the conference progressed, I did become convinced that standards were in fact going to happen (Bredekamp page 77), that the people at the conference should be involved in writing them (although I wished more voices were included, particularly of those who were underrepresented or not represented at the conference: child-care providers, people of color, low-income people, people speaking as parents, as representatives of community groups, etc.), and that early childhood standards are worthwhile in themselves. I am not convinced, however, that such standards won’t be misused: although the recommendations seek to “…avoid the abuses and realize the advantages of specifying goals,” (page 9), the authors can’t control whether or how others use the goals set out in this book.

The central paradox of standards is that the more specific they are, the easier they are to both use and misuse. Specific mastery goals that might start out as a means to implement a vision of excellence can become ends in themselves. For example, in April of 2002, President Bush proposed denying funding to Head Start programs that failed to teach children the alphabet and how to count to ten.4 This proposal epitomizes a narrow, rigid standard that is easy to measure and that, although important for children to meet can lead to program decisions that circumvent understanding and ask children to perform rote, especially when funding is at stake. This book includes many recommendations that are mastery goals, just not “narrow and rigid” ones (page 9). As Bredekamp says, “each of them [the potential disadvantages of standards] is based on the same assumption: that the mastery goals set will be the wrong goals (page 78).” Even if the mastery goals are the “right” goals, however, they can still be misused.

The charge of the conference and this book was to develop a consensus document supported by many different “stakeholders” however, it is important not to let the desire for consensus obscure very real differences in agenda. I will not try to sort out all the multiple agendas involved in mathematics education reform efforts and the complex ways in which they intersect; despite large areas of common ground, agendas sometimes contradict each other, and some agendas are not centered on the best interest of children. For example, many of us see citizenship as a central motivation for educational reform – to give children the mathematics background and the thinking skills they need to create a more informed citizenry that can think critically about social and political issues, but
politicians and businesses that regularly use misleading quantitative information to convince people to buy their products or support their policies are also “stakeholders.”

The authors include many caveats and statements of intent to try to prevent misuse; even more important to this effort is working to insure that a widespread community of people, including teachers, parents, and policymakers, support and understand the goals and appropriate uses of the standards, as only one facet of an effort to improve early childhood education in mathematics. Despite my reservations, I do think this book makes a compelling case for standards, when carefully defined, as a way to improve mathematics education for young children.

The book includes four major justifications for standards: they can help demystify what children can do, help teachers focus learning on important knowledge and skills, help parents better understand their children’s mathematics learning, and promote equity (pages 9-10). I find the first three justifications self-evident: because mathematics has not traditionally been a focus in early childhood education (other than in narrow ways such as learning to count or identify shapes (Clements, page 268)), teachers and parents often don’t know what is possible and what is important with regards to mathematics education, and standards have the potential to help this situation, as long as they are the “right” standards, used for the right purposes (Bredekamp, Chapter 2).

I would like to look a little closer at the fourth justification for standards, that they promote equity. Recommendation 1 begins (page 13),

> Equity is a major concern in mathematics education at all levels. There is an early developmental basis for later achievement differences in mathematics: children from different socio-cultural backgrounds may have different foundational experiences.

In other words, equity is a major concern because there is inequity, with the unstated assumption that inequity is wrong, that is, such inequity is unjust. Thus, the fundamental basis for promoting equity is justice.

Many arguments for equity in mathematics education focus on jobs and economic and national security, not on justice. Of course, economic access is a large component of social justice, but I am not convinced that the “vast majority” of jobs in a global economy require more sophisticated skills than in the past (page ix). Certainly many jobs in the U.S. that currently afford a decent standard of living require such skills, but there are still plenty of jobs in restaurants, sweatshops, hotels, and industrial farms that don’t require much mathematics and that need to be done, and at the current moment, there are also many unemployed, mathematically literate, people. An argument for equity in education that focuses primarily on our nation not having a skilled enough work force is an argument for access to high quality mathematics education for more children, not for all children, and of course, it is low income children from historically oppressed groups who are most likely to be excluded.

A justice-based vision of excellence and equity says that all children should be able to study mathematics for the same reasons outlined earlier – because it is intrinsically interesting, to develop thinking skills, and to help understand and participate in the world.
Not only does a justice-based vision of excellence give children tools for jobs, it also gives them tools for citizenship and democratic participation. A justice-based vision of excellence and equity is not exclusive to mathematics: all children should have access to high quality education in all subjects.

I’m fairly certain that most everyone at the conference would agree with such a justice-based vision of excellence and equity, and implementing such a vision of must move far beyond the mathematics classroom. Inequity in mathematics education is only one part of the social injustice confronted by children living in poverty; math standards won’t address malnutrition, inadequate housing, lack of access to health care, violence, and other factors that interfere with learning, yet I think it important that we view mathematics standards as one part of an interconnected movement for justice for children and their communities.

Mathematics Standards

The book’s three criteria for mathematics curriculum standards (page 10 and 11) are: guidelines should be based on available research and expert practice, guidelines should be focused on the “big ideas” of the mathematics of children, and guidelines should represent a range of expectations for child outcomes that are developmentally appropriate. I found the explanation of the “big ideas” of early childhood mathematics (pages 15-57) to be particularly interesting and helpful (especially the charts on pages 16, 18, 39, and 50). Much of the information was new to me, and it reflects part of my general feeling of the conference: that mathematics for young children is a lot more interesting than I had previously realized. Since the conference, I’ve had the opportunity to talk with many kindergarteners and first graders about mathematics, and these conversations have only enhanced my interest. I don’t have much more to say about the criteria that standards be based on big mathematical ideas and that they be developmentally appropriate; the only areas for debate here are the specifics of what constitutes a “big idea” or what is “developmentally appropriate.”

However, I do have more to say about the book’s first criterion for mathematics standards, that they should be based on available research and expert practice. This recommendation makes sense, but needs to be read in the context of the limitations of education research. I write this as one with a mathematician’s aesthetic: I like it that in mathematics we can prove things with a standard that is not possible in educational research. Nonetheless, it is critical that we are both informed by research and that we maintain the utmost integrity about what research does and does not tell us; misrepresenting the limitations of available research contributes to mathematics and science illiteracy and can easily backfire.

The use of research takes place in a political context. For example, The National Reading Panel, a blue-ribbon commission on reading instruction convened by President Bush presents its work as determining “scientifically based reading instruction,” using a definition of a “scientific” study that was biased towards phonics as a method of instruction. Determining how children learn and what educational methods are most successful in helping them learn is a complicated, messy process that never takes place in
a neutral, apolitical context; it is not a matter of, as commonly reported in the media, “experts proving facts.”

In an earlier draft of his article, Baroody (Chapter 8) reviewed the way psychologists’ views of young children’s mathematical competencies have changed over the course of the twentieth century, moving from a pessimistic view to an extremely optimistic view to a middle ground view. This type of research, usually conducted with one child and one researcher, is the basis for many of the recommendations on mathematics standards in this book; a major difficulty with this type of research is insuring that the study shows what it purports to show; both bias on the part of the researcher and problems with controlling for extraneous variables can lead to misleading results, especially when researchers must interpret the responses of infants and young children. A good example of potential misinterpretation in such studies arose at the conference in a discussion of the research of Mix and colleagues, as reported in Baroody (page 211). The researchers showed that four and five year old children were significantly better than three year olds at determining the correct result of combining portions of a circle; initially, Baroody presented this work as showing that the children understood how to add fractions, but others thought that the children could have been responding to perceptual clues about area; Baroody suggests further research (page 211).

It can be easy to miss subtleties and misrepresent what educational research has actually measured. At the conference I was quite taken with the presentation on Seo and Ginsburg’s research (Chapter 4), where they videotaped young children during free play and then analyzed the tapes to determine how much time children spent on various mathematical activities. I remembered the presenter as saying that almost half of children’s free play time is spent on mathematical activities; I eagerly shared this information with my students, especially the prospective teachers who still thought mathematics for children is, “plus, minus, times, divide.” Upon reading the article in this volume more carefully, however, I realized that I did not report this information accurately; the authors explain that, “The results showed that children exhibited at least one mathematical activity during an average of 43% of the minutes,” (page 95) which is different than saying the children spent almost half their total time on mathematical activities. For example, if the children spent 6 seconds during each of the 43% of the minutes on a mathematical activity, they would have spent only 4.3% of their time on mathematical activities. I presume that this latter interpretation is exaggerated, yet I did misinterpret the research. The authors of the study made a choice on how to interpret the videotapes; had they made another choice, different misinterpretations would have been possible. I’m sure the presenter was very clear, but in my enthusiasm for new information, I missed important details.

The research on children’s learning underlying this book’s recommendations for mathematics standards, particularly with regard to number and operation, seems solid. As tricky as this research can be, it is largely one-on-one and includes far fewer variables than research on children’s learning in actual classrooms, and it takes place in the context of a vibrant community willing to challenge and revisit results. The strengths of this research are, however, also its limitations: a laboratory situation is simply not the same as a classroom situation, and laboratory research doesn’t necessarily carry over to settings involving a group of children interacting with each other, with a teacher, with a
curriculum, and with all the idiosyncrasies of a particular setting. Also, research on children’s learning is not the same as research on teaching; a clear consensus on developmental trajectories in learning a particular topic does not imply a clear consensus on how to best facilitate such learning. Educational research is an enormously complex field: I am far from an expert and am merely trying to highlight some of the complications.

The first criterion for guidelines for mathematics standards also states that standards should be based on expert practice, as well as on research; however, this book discusses research much more than expert practice. Figuring out how to identify and effectively implement expert practice is yet another complex problem; what works for an expert might look different when tried by a novice. Besides conducting more research, especially on geometry (this book includes little research on children’s concepts of geometry in three-dimensions, especially problematic in light of Seo and Ginsburg’s results), patterns, probability, data analysis, the use of technology, and other under-researched topics, the next step for developing standards should include ways to better honor, include, and transmit expert practice.

Recommendations for mathematics standards (Recommendations 2-4) include that standards for programs, teaching, and assessment are most important; that mathematics for young children should be an integrated whole; and that processes are as important as content. Expert practice is certainly as important as research in developing programs that work, in designing curricula that integrate mathematics with other subjects and subjects within mathematics, and in fostering problem solving, communication, and other processes.

**Curriculum, Learning, Teaching and Assessment**

Doug Clements’ remarks about geometry in the elementary curriculum left a big impression on me after the conference; this information is included in this book, where he describes how geometry curricula for young children usually consist of identifying four shapes, and only in particular proportions and orientations. Teachers’ dialogue with children about geometry is often exceptionally superficial (page 268). Coupled with Seo and Ginsburg’s results observing pattern and shape as the mathematical activity that the children they observed engaged in most in free play (page 90), it seems obvious that major improvements in geometry curricula can and should be made. Changing the geometry curriculum is an example of many of the purposes of this book: it’s based on research about both the status quo and children’s often untapped capacities; it involves both professional development and new materials; it makes use technology; and it can make a difference in later understanding. At the conference I was also struck by the information on the Building Blocks Curriculum (Chapter 15); when I first heard of virtual manipulatives, I thought they sounded like silly gimmicks, but I changed my mind at the conference and have since used such computer manipulatives with many students.

There’s a big split in this book between discussion of curriculum, learning, and assessment for pre-school and for school, with kindergarten sometimes included in one or the other; the split makes sense since even diehard traditionalists have different expectations for pre-school than for school. The first section of this book addresses pre-
school exclusively, and a major concern is that in adding more mathematics content, programs not lose the advantages of the pre-school culture and gain the disadvantages of the traditional school culture. As Lindquist and Joyner say, “Can we balance higher expectations with the aim of preschool to foster a love for learning, a feeling of success, and the joy of being a child?” (page 450). Mathematics has not been central to preschool programs in the past, and it’s important that adding new mathematics content not mean thoughtlessly pushing down curriculum, that may in fact not even be appropriate for older children (page 450).

Recommendations 6-12 on Curriculum, Teaching, and Learning reflect a desire to maintain the positive values of the preschool culture (pages 58-63): mathematical experiences should build on play, teachers’ most important role is to help children reflect and structure environments that support mathematical activities, teachers should use a variety of strategies to promote children’s learning, children benefit from thoughtful use of technology, teachers should endeavor to understand each child’s mathematical ideas, teachers should help children develop strong relationships between concepts and skills, and the primary goal assessing young children is to understand their thinking and to inform teaching. The materials and curriculum included in this book also seem to follow these recommendations and fit within the pre-school culture (Chapters 13-16).

The only recommendation for this section not mentioned above is Recommendation 5, that approaches to teaching, learning, curriculum, and assessment be based on research and should be developed and tested extensively with children. The spirit of the recommendation – that curricula should be designed thoughtfully, tested with diverse groups of children, and undergo cycles of revision – make perfect sense, but, as discussed earlier, research on implementing curricula is complicated. In practice, teaching is not a laboratory science: there are far too many variables involved to control for all-but-one, as is even difficult in the laboratory setting. The text (page 58) indicates that more sophisticated models of developing new curricula exist; such models should certainly be used, but, once again, not in an uncritical manner that simply designates a particular curriculum “scientifically proven” to work.

In their study, Klein and Starkey (Chapter 14) seem to have controlled for many variables when piloting their new curriculum. For their controls, they used children who had been taught by the same teacher with a different curriculum in the year before, and they compared these children’s mathematics achievement to that of children who used the new curriculum. The new curriculum led to impressive results, particularly in helping children from low-income homes catch up with children from middle class homes in mathematics achievement. The study also involved extensive professional development for the teachers, and it’s impossible to separate the effects of the professional development from those of the curriculum itself (and from other reasons the second group could have done better, such as random fluctuations from one class to another or teachers altering their practice simply because they were in a study). Assuming the program was effective, there are many problems that can happen when such a project is attempted on a larger scale: corners are often cut, professional development can be scaled back, and the program implemented can become a different program than the one supported by the research.
Catherine Sophian states that, “The modes of instruction that produce the greatest immediate learning may not always be the ones that are best for the long term.” (page 253). She then gives a detailed and convincing argument that the ways we usually teach whole number concepts interfere with learning fraction concepts later on. She advocates a “prospective developmental perspective” that takes the long-range goals of mathematics education into account (page 265) when planning curricula and pedagogical approaches. Implicit in her argument is an argument that teachers need to know mathematics well beyond what they are actually teaching. The prospective developmental perspective is an important concept; however, it seems more complicated to apply it to some other topics besides fractions. For example, “Big Idea 2.1” (Baroody page 193) states “A collection can be made larger by adding items to it and made smaller by taking some away from it.” This “big idea,” which is absolutely essential for young children, has great potential to interfere with children’s later conceptualization of negative numbers, where addition can make things smaller and subtraction can make them bigger.

When the book’s discussion turns to school education, two authors continue to agree on the centrality of making mathematics meaningful to children, but they approach the issue of curriculum from very different angles. Karen Fuson (Chapter 5) starts from the premise that the U.S. already has a de facto national mathematics curriculum, widely criticized as “a mile wide and an inch deep,” set by the textbook companies, and heavily influenced by the need to gain approval in large states. These states also have low expectations for their many non-English speaking children and children living in poverty. She then proposes a different curriculum, one organized into coherent “chunks,” which she supports using international data and research on effective curricula. Much of her discussion focuses on ways of “achieving mastery for all,” grounded in the reality of the classroom. If implemented, Fuson’s proposal would be a huge improvement over the status quo.

Les Steffe starts by pointing out an “unavoidable” tension in PSSM between honoring that children’s ways of thinking about mathematics are different from adults’ and insisting that children meet age-specific benchmarks set by adults (page 222). The first part of this book reflects a conscious decision to report all age-related benchmarks solely as developmental trajectories (Tables 1.1, 1.2, and 1.3); however, as children get older, the school culture includes many more external benchmarks than the pre-school culture. Steffe talks of “children’s mathematics” as the mathematics that children have constructed before the teacher attempts to teach them and the “mathematics of children” as the models teachers make to explain children’s actions (page 224). He asks what it would mean for teachers to assume that the mathematics of children is rational and to center their practice on learning to interact mathematically with children to bring forth, sustain, and modify their children’s mathematical knowledge, rather than simply trying to transfer their knowledge to the children (page 235).

I found myself largely agreeing with both Fuson’s and Steffe’s ideas, yet struggling to reconcile them, given that they largely contradict each other, with Fuson calling for a national curriculum and Steffe seeing individual teachers and children as co-authors of curriculum. I read Steffe from my point of view as an individual teacher, who teaches of classes full of individual students, and in his work I recognize some of my own struggles to learn to communicate more effectively with my students – to understand their
mathematical thinking at a given time and to make instructional decisions that will best support their learning. I find it difficult to imagine extrapolating both Steffe’s ideas and my own classroom experience to the scale of which Fuson describes, however, and thus Fuson’s ideas make sense as a radical, large-scale step that looks at the country as a whole and engages with current political reality and how it might be advanced.

Such tensions between large-scale and small-scale views pervade discussions of curriculum, learning, teaching, and assessment. Mathematics education reform is an attempt to implement large-scale changes in small-scale environments. Research is critical at every scale – from the individual teacher to the classroom to the district to the state and to the nation -- but such research needs to be examined carefully to determine its relevance both laterally and at different scales.

4. Professional Development

Effective professional development for early childhood teachers is clearly one of the most important components in improving mathematics education for young children. Teachers are affirmed throughout this report as playing the most critical role in creating a high-quality educational environment (page 59). To be more effective at teaching mathematics, early childhood teachers need to know more about mathematics, children’s mathematical thinking, and how to work with children in mathematics (page 66-67). As a group, early childhood teachers generally did not have good experiences with their own mathematics education, some even chose to work with younger children in order to avoid teaching more advanced mathematics (Copley page 401). Furthermore, the teachers and caregivers for many pre-kindergarteners are not in school settings (Copple page 83), where it’s even more difficult to organize professional development opportunities.

Sarama and DiBiase (Chapter 19) provide a comprehensive overview of research on professional development, beginning with a discussion of some of the serious issues involved in trying to conduct such research and implement its results into practice, especially vexing is trying to define and measure the effectiveness of professional development (page 417); there are also considerable economic and institutional barriers to implementing professional development (page 418). Beyond the practical and methodological considerations, effective professional development seeks to change teacher’s common beliefs that, “mathematics is a set of facts and that memorizing these facts is an appropriate route to learning mathematics” (page 419), and changing such beliefs requires that teachers be dissatisfied with their existing beliefs, find the alternatives intelligible and practical, and connect new beliefs to old conceptions (page 420).

Sarama and DiBiase go on to analyze research on professional development in general, in early childhood education, and in mathematics education; they look at successful professional development programs, and they end with a list of recommendations (page 438-439). One theme of the research is that successful professional development needs to be “an ongoing activity woven into the fabric of their [teachers’] professional lives, including a focus on curriculum, assessment, leadership, and collegial sharing” (page 423). Many of the most widespread kinds of professional
development are just the opposite: one-shot workshops, new curricula implemented without sufficient support, and professional development that is imposed on teachers, that doesn’t “start where they are at,” and that teachers perceive as abstract and disconnected from their practice.

Both Rachelle Feiler’s (Chapter 17) and Juanita Copley’s (Chapter 18) articles show examples that reflect many of the themes Sarama and DiBiase discuss. Feiler starts from what teachers already value and do well, and Copley proposes a wonderful model for professional development that involves teachers, undergraduate pre-service teachers, and children, with a great deal of respect and learning involved for all.

Sarama and DiBiase make some small references to pre-service teacher education (page 419 and 423), in the context of its ineffectiveness for preparing early childhood teachers in mathematics, and in general this book doesn’t give it much attention (Baroody (pages 149-150) and Copley (Chapter 18) include some discussion), but it merits a closer look. Many early childhood teachers are not graduates of teacher education programs, but many are, and improving such programs is a more contained, well-defined task than implementing wide spread professional development for current teachers.

The recently issued report The Mathematical Education of Teachers, from the Conference Board of the Mathematical Sciences6, is directed primarily toward mathematicians. It includes much discussion of the mathematical content and pedagogy of elementary mathematics and the features of high-quality courses for prospective K-12 teachers. It recommends that prospective elementary teachers take at least nine semester hours of high-quality courses on fundamental ideas of elementary school mathematics and that such courses should be taught by mathematicians who have a serious interest in teacher education, and who work closely with faculty in mathematics education (pages 7-8). Its vision is fully compatible with this book. A new national project, “Preparing Mathematicians to Educate Teachers” is also working to provide necessary professional development for mathematicians new to working with teachers; so far, the majority of workshops for this project have been on working with prospective elementary teachers.

Teacher education needs to include content, pedagogy, and mathematical psychology (Baroody page 150), and I would argue that good teacher education must also include good liberal arts and sciences education. Too many education students enter college with a love of children, but without a love of learning. Returning to the reasons for studying mathematics given earlier, prospective teachers need to be exposed to mathematics as a fascinating subject that some people love, not necessarily so that they will love it themselves, but so that they will see what it is. Prospective teachers need to develop their own critical thinking skills. Prospective teachers need to develop quantitative literacy, which they should encounter throughout the curriculum, not just in mathematics courses, so that they can read research critically, better add their voices to civic debate, apply mathematics in their own lives, and become role models as adults who are comfortable with mathematics.7
While I also agree that there is a strong need for mathematics courses that are

Too many education students enter college with a love of children, but without a love of learning embedded in practice (Sarama and DiBiase page 424), my work on-site with graduate interns at a K-8 school reconfirmed for me the value of mathematics content classes for prospective teachers (of course, with a pedagogy that emphasizes understanding and attention to problem solving, reasoning, and other processes). While the on-site work clearly helps make connections between mathematics understanding and practice, it was difficult to help new teachers, dealing with classroom management and many other practical concerns, improve their content understanding. I saw very clear differences in understanding between undergraduates who have been through a rigorous year-long mathematics program and these graduate students who were expected to develop their understanding primarily in context. I could also see, however, that even the interns whose mathematics understanding was reasonably solid, still needed a lot of support to learn to use that knowledge to understand children’s thinking, to ask appropriate questions, and to make good instructional decisions, and thus no matter how good the pre-service program, ongoing professional development is absolutely necessary.

One issue that seems critical to me in the discussions of professional development is respect. Calls to implement better professional development come with an agenda that is at least mildly coercive: seeking to change teachers’ beliefs, knowledge, and practice, because someone else knows better. Of course, I think that those of us who, for example, see mathematics as more than a set of facts to memorize, do, in fact, “know better,” at least on this particular issue, and thus at some level we do think a teacher who believes mathematics is just number facts and shape names is wrong, although we might understand how s/he developed that belief. Nonetheless, people leading professional development often come with more academic and professional status, better salaries, and often race and class privilege, into a situation where previous professional development efforts often were fundamentally disrespectful – of teachers’ time, knowledge, and wishes. Thus, the call in Recommendation 13 (page 66), that professional development emphasize the importance of teacher leaders and collegial support groups is one part of working towards a respectful environment, but the issue is by no means simple.

5. Implementation

The issue of implementation merits at least another conference and book. Setting standards is only a very first step -- connecting standards to curriculum and teaching is the necessary work of implementing a vision of excellence for all children in mathematics. I see three different types of thinking about implementation in this book. One form of implementation is large-scale dissemination of small amounts of very accessible information and resources, including things like pamphlets and public service announcements, resources for parents, toys, etc. (page 453). Such dissemination has a lot of potential. There is much basic information about how children learn mathematics that is not widely known; a lot of education can be done at the beginning of the learning
curve. Such efforts have potential to reach child-care providers and others who are not likely to be served by professional development opportunities.

Another way of implementing standards is represented by the many projects to improve curriculum and professional development reported in this book. This level of implementation also mediates between a vision and current reality, but with a longer-term focus on implementing (sometimes measurable) change; it also includes working for policy changes.

The final level of implementation involves creating the large-scale changes necessary to truly support this book’s vision of excellence in education for all children. This form of implementation requires many voices and must include those most harmed by our country’s inequities. The last sentence of the last recommendation of this book (page 69) reads:

Governments should provide adequate funding and structures so as to provide high-quality early childhood education for all children, including high quality professional development for the adults who care for them.

This recommendation, tucked neatly at the end of part 1 of the book, is actually a call for major social change. On behalf of all children, and the adults who care for them, we should settle for nothing less.

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