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Technology in Early Childhood Mathematics: *Building Blocks* as an Innovative Technology- Based Curriculum

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Building Blocks is a new Pre-K to Grade 2, software-enhanced, mathematics curriculum designed to comprehensively address the year 2000 standards of the National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics*. In this chapter, I describe the basic features of the *Building Blocks* program, including the research on which it was based, and specific activities from the program, including the results of field tests of those activities with a range of teachers and children.

THE DESIGN OF *BUILDING BLOCKS*

Building Blocks is designed to enable all young children to build solid content knowledge and develop higher-order, or critical, thinking. To achieve this, we (Douglas H. Clements and I) needed to consider the audience, determine the basic approach to learning and teaching, and draw from theory and research in each phase of the design and development process. In this section, I briefly overview these three areas; I describe them in more detail in the following sections.

The demographics of the early end of the age range imply that materials should be designed for home, day care, and classroom environments, and

for children from a variety of backgrounds, interests, and ability levels. To reach this broad spectrum, the materials are progressively layered—users will be able to “dig deeper” into them to reach increasingly rich, but demanding, pedagogical and mathematical levels.

Based on theory and research on early childhood learning and teaching (Bowman, Donovan, & Burns, 2001; Clements, 2001), we determined that *Building Blocks*' basic approach would be *finding the mathematics in, and developing mathematics from, children's activity*. The materials are designed to help children extend and mathematize their everyday activities, from building blocks to art to songs and stories to puzzles. Activities are designed based on children's experiences and interests, with an emphasis on supporting the development of *mathematical* activity. So, the materials do not rely on technology alone, but integrate three types of media: computers, manipulatives (and everyday objects), and print.

Many claim a research basis for their materials, but these claims are often vacuous, citing vague theories without specifics (Sarama & Clements, in press). *Building Blocks* is research based in several fundamental ways. Our design process is based on the assumption that curriculum and software design can and should have an explicit theoretical and empirical foundation, beyond its genesis in someone's intuitive grasp of children's learning. It also should interact with the ongoing development of theory and research—reaching toward the ideal of testing a theory by testing the software and the curriculum in which it is embedded. Our model includes specification of mathematical ideas (computer objects) and processes/skills (computer tools) and extensive field-testing from the first inception through to large summative evaluation studies (Clements, 2002; Clements & Battista, 2000; Sarama & Clements, in press). Phases of this nine-step design process model are: drafting curriculum goals, building an explicit model of children's knowledge and learning in the goal domain, creating an initial design, investigating components of the software design, assessing prototypes and curriculum (with one-on-one interviews with students and teachers), conducting pilot tests (in a few classrooms), conducting field tests in numerous classrooms, and publishing the materials. All the while, feedback from the field results in further refinement to the design of the software and activities, which then results in further testing. In this way, we continually loop through the earlier phases of the model.

Several steps deserve a bit more elaboration. The step of “building an explicit model of children's knowledge and learning in the goal domain” involves the adaptation, creation, and use of learning trajectories. *Building Blocks* is structured on empirically based learning trajectories through the big ideas and skill areas of mathematics (Clements & Battista, 1992; Fuson, 1997). The step of “creating an initial design” is based largely on these learning trajectories (which are discussed in detail in Part I and several chapters of this volume), but also on other bodies of research. For example, what mathematics is included is based on research on what topics are developmentally appropriate for, generative for, and interesting to young

children. As another example, the design directly applies research on making computer software for young children motivating and educationally effective (Clements, Nastasi, & Swaminathan, 1993; Clements & Swaminathan, 1995). It is to this last issue that I next turn.

DEVELOPMENTALLY APPROPRIATE PRESCHOOL MATHEMATICS SOFTWARE

The extensive use of software in *Building Blocks* requires mining the existing research for what it tells us about young children's use of, and learning from, computer programs. Although the basic question of whether computers are "developmentally appropriate" for young children at all is still debated, I do not discuss it in detail, as the research is clear that, *used wisely*, computer use can be meaningful, motivating, and beneficial for children 3 years of age and above (Davidson & Wright, 1994; Haugland, 2000; Haugland & Wright, 1997; Sarama & Clements, 2002; Shade, 1994).

The research also indicates that not all uses of computers are valuable and that teachers must work hard to integrate technology effectively (Wright, 1998). As an example of the first finding, although hundreds of products are now available for young children that include mathematics, most of these products fall into one of three categories (Clements & Nastasi, 1992; Sarama & Clements, 2002). The first category is drill programs, often disguised by multimedia "bells and whistles." Such programs can be effective at their intended purpose, providing practice, but they do not develop conceptual knowledge. Too often, the drill in these packages is not optimally designed (e.g., not using the computer's management capabilities to provide targeted practice). The second, related, category, often called "Edutainment," also has attractive multimedia features, but limited mathematics content and pedagogy. The third category, exploratory environments, has potential for mathematical investigations, but unfortunately young children usually explore them only on the surface level. For both the second and third categories, there is little learning, by children or educators (Sarama & Clements, 2002).

In contrast, programs with focused goals and coherent pedagogy can help young children develop both concepts and skills (Clements & Nastasi, 1992; Sarama & Clements, 2002). For example, the flexibility of computer technologies allows the creation of a vision not limited by traditional materials and pedagogical approaches (cf. Confrey, 1996). As just one illustration, computer-based communication can allow representations and actions not possible with other media. As an example of the latter, blocks can be actually glued together, trucks can leave paths in the sand, and these paths can be changed, moved, saved, and used later with other vehicles. The *Building Blocks* materials not only ensure that the actions and objects mirror concepts and procedures, but also that they are embedded in tasks and developmentally appropriate settings (e.g., narratives, fantasy worlds, building projects).

The second research finding is that guidance by the teacher is essential for effective integration of technology. Teachers must introduce, monitor, and mediate children's interactions with computer programs (Clements & Nastasi, 1992; Sarama & Clements, 2002). *Building Blocks* integrates technology activities with off-computer activities, and helps the teacher provide such mediation by providing pedagogical guidance for both off- and on-computer activities in the teachers' materials.

We designed *Building Blocks* within this general research framework. We also looked at the research on particular uses of computers to teach mathematics to young children. One decision was to use computer manipulatives. Some early childhood educators may argue that young children benefit much more from the tactile experience of interacting with concrete manipulatives. But can on-screen manipulatives still be "concrete"? One has to examine what *concrete* means. Sensory characteristics do not adequately define it (Clements & McMillen, 1996; Wilensky, 1991).

First, it cannot be assumed that children's conceptions of the manipulatives are similar to adults' (Clements & McMillen, 1996). For example, a student working on place value with beans and beansticks used the bean as 10 and the beanstick as 1 (Hiebert & Wearne, 1992). Second, physical actions with certain manipulatives may suggest different mental actions than those we wish students to learn. For example, researchers found a mismatch among students using the number line to perform addition. When adding five and four, the students located 5, counted "one, two, three, four," and read the answer. This did not help them solve the problem mentally, for to do so they have to count "six, seven, eight, nine" and at the same time count the counts—6 is 1, 7 is 2, and so on. These actions are quite different (Gravemeijer, 1991). Thus, manipulatives themselves do not carry the meaning of the mathematical idea. Students must act on these manipulatives in the context of well-planned activities, and ultimately reflect on these actions. Later, we expect them to have a "concrete" understanding that goes beyond these physical manipulatives.

It appears that there are different ways to define *concrete* (Clements & McMillen, 1996). We define Sensory-Concrete knowledge as that in which students must use sensory material to make sense of an idea. For example, at early stages, children cannot count, add, or subtract meaningfully unless they have actual things. They build Integrated-Concrete knowledge as they learn. Such knowledge is connected in special ways. This is the root of the word *concrete*—"to grow together." What gives sidewalk concrete its strength is the combination of separate particles in an interconnected mass. What gives Integrated-Concrete thinking its strength is the combination of many separate ideas in an interconnected structure of knowledge (Clements & McMillen, 1996).

For example, computer programs may allow children to manipulate on-screen pattern blocks. Children working with physical pattern blocks may develop sophisticated ideas of symmetry and geometric relationships, but research has shown they often do not (Sarama,

Clements, & Vukelic, 1996). The computer manipulatives offer several mathematical and practical benefits. The software encourages explicit awareness of the geometric motions used in creating a design. Specific tools can allow children to dynamically explore composition and decomposition of shapes. The "flatness" of the on-screen manipulatives facilitates exploration of relationships between shapes (e.g., Matthew became frustrated working off-computer after working with the software because he was unable to cover half of his blue rhombus to get a much desired blue triangle).

Practical benefits include being able to easily move a design when more space is needed. Off-computer, moving the physical manipulatives often results in the design falling apart. On-computer, children can glue the shapes together before quickly sliding the entire design to another part of the screen. Similarly, most children simply destroy their pattern block designs when it is "clean-up" time. Children working on computer can save their work and later come back to reflect, reproduce, or extend.

Computers encourage students to make their knowledge explicit, which helps them build Integrated-Concrete knowledge. Specific theoretically and empirically grounded advantages of using computer manipulatives include (Clements & McMillen, 1996): providing a manageable, clean manipulative; offering flexibility; changing arrangement or representation; storing and later retrieving configurations; recording and replaying students' actions; linking the concrete and the symbolic with feedback; dynamically linking multiple representations; changing the very nature of the manipulative; linking the specific to the general; encouraging problem posing and conjecturing; scaffolding problem solving; focusing attention and increasing motivation; and encouraging and facilitating complete, precise explanations.

Of course, multimedia and other computer capabilities should be (and, in *Building Blocks* are) used when they serve educational purposes. Features such as animation, music, surprise elements, and especially consistent interaction get and hold children's interest (Escobedo & Evans, 1997). They can also aid learning, *if* designed to be consistent with, and supporting, the pedagogical goals. In addition, access to technology is an important equity issue. We plan on having much of our materials running on the widely available Internet (Word Wide Web).

In summary, we designed the *Building Blocks* project to combine the art and science of teaching and learning with the science of technology, with the latter serving the former. Such synthesis of (a) curriculum and technology development as a scientific enterprise and (b) mathematics education research will reduce the separation of research and practice in mathematics and technology education. This will produce materials based on research and research based on effective and ecologically sound learning situations. Moreover, these results will be immediately applicable by practitioners (parents, teachers, and teacher educators), administrators and policymakers, and curriculum and software developers.

MATHEMATICAL CONTENT

What mathematics should we teach? Basic mathematics for preschool children can be organized into two areas: (a) geometric and spatial ideas and skills and (b) numeric and quantitative ideas and skills. Research shows that young children are endowed with intuitive and informal capabilities in both these areas (Bransford, Brown, & Cocking, 1999; Clements, 1999a). Three mathematical themes should be woven through both these main areas: (a) patterns, (b) data, and (c) sorting and sequencing. These are children's *mathematical building blocks*, or ways of knowing the world mathematically. We illustrate one approach to developing these concepts through our *Building Blocks* project.¹

As stated, we believe a good mathematics curriculum is based on *finding the mathematics in, and developing mathematics from, children's activity*. In this approach, children extend and mathematize their everyday activities, from art to songs to puzzles to, of course, building blocks (this is another meaning behind the *Building Blocks* name). So, we designed activities based on children's experiences and interests, with an emphasis on supporting the development of *mathematical activity*. Mathematization emphasizes representing—creating models of activity with mathematical objects, such as numbers and shapes, and mathematical actions, such as counting or transforming shapes. Our materials embody these actions-on-objects in a way that mirrors what research has identified as critical *mental actions*—children's *cognitive building blocks* (the third meaning of the name). These cognitive building blocks include creating, copying, and combining objects such as shapes or numbers. A following section illustrates how such actions-on-objects are embedded in the activities.

ROLE OF THE COMPUTER

An illustration of mathematical actions-on-objects, and how they are embodied in both off-computer and computer activities, builds on young children's experiences with and love of puzzles. This set of activities is briefly described in Fig. 15.1. Children fill in outline puzzles using an extended set of pattern blocks. The *objects* they use are basic geometric shapes. The *actions* they perform on these objects include sliding, turning, flipping, and combining or composing. They initially solve outline puzzles with physical pattern blocks off the computer (Fig. 15.1a). They solve similar puzzles at the computer, enjoying that the blocks "snap" and stay together accurately (Fig. 15.1b). More importantly, they use the computer's tools to perform actions on the shapes (Fig. 15.1c). Because they have to figure out how to move the blocks and *choose* a motion such as

¹"Building Blocks—Foundations for Mathematical Thinking, Pre-Kindergarten to Grade 2: Research-Based Materials Development." is being developed at the State University of New York at Buffalo and Wayne State University, where Julie Sarama is the codirector. See <http://www.gse.buffalo.edu/org/buildingblocks/>.

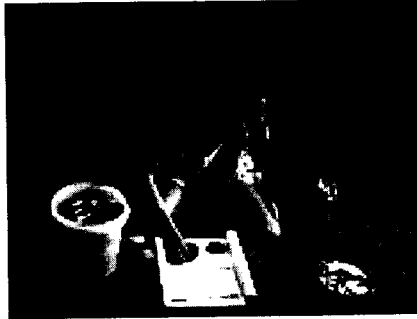


FIG. 15.1a. Children solve puzzles by filling a puzzle outline with pattern blocks. They solve each one in several different ways if they are interested.

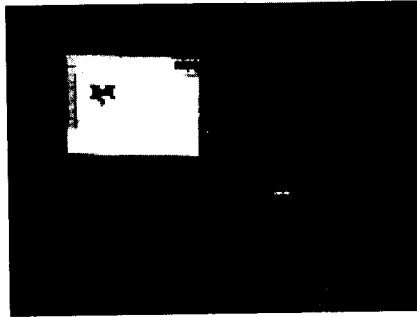


FIG. 15.1b. Then children do a similar activity on the computer, where the pattern blocks snap together in a satisfying manner and stay put.

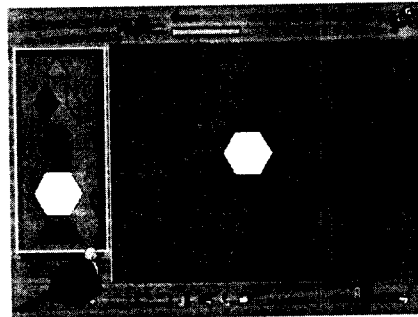


FIG. 15.1c. Also, the computer's tools for actions such as sliding and turning help children become more aware of these mathematical processes.

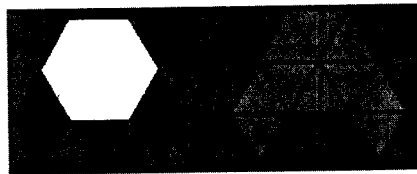


FIG. 15.1d. Children might be challenged to find a way to use the fewest (or the most) blocks to fill an outline.

FIG. 15.1a-15.1d. The "Shape Puzzles" activity.

slide or turn, they are more conscious of these geometric motions. Four-year-old Leah initially referred to the “spinning” tools, but later called them the “turn shapes” tools, and after several months was describing directions and quantities, such as “OK, get *this* [right or clockwise] turn tool and turn it three times!” Such choices also encourage children to be more *deliberate*. They “think ahead” and talk to each other about what shape and action to choose next. In these ways, the computer slows down their actions and increases their reflection. Just as important, using the motion tools deliberately helps children become familiar with seeing shapes in different orientations and realizing that changing the orientation does not affect the shape’s name or class. In a related activity, children are challenged to build a picture or design with physical blocks and copy it onto the computer. Again, this requires the use of specific tools for the geometric motions of slide, flip, and turn and encourages children to reflect on the orientation of the shapes. Children who are experienced or advanced may be challenged in several ways. Some might enjoy finding a way to use the fewest blocks to fill an outline (Fig. 15.1d).

How are learning trajectories embodied in this activity? Our theoretical learning trajectory (see Clements, chap. 10, this volume) guides the selection of puzzles for children at different levels of the trajectory. For example, the puzzle in Fig. 15.2a would be presented to the child at the level of *Pre-Composer*, whereas the puzzle in Fig. 15.2b would be presented to the child at the level of *Picture Maker* (just to be sure you know what it is, Fig. 15.2c shows the final picture), and the puzzle in Fig. 15.2d would be presented to the child at the level of *Shape Composer*.

These are powerful mathematical processes to perform on shapes, but not out of the reach of preschoolers (although some will reach only the lower levels). Research shows that preschoolers know a considerable amount about shapes (Clements, Swaminathan, Hannibal, & Sarama, 1999; Lehrer, Jenkins, & Osana, 1998), and they can do more than we assume, especially working with computers (Sarama et al., 1996). In the broad area of geometry and space, they can do the following: recognize, name, build, draw, describe, compare, and sort two- and three-dimensional shapes, investigate putting shapes together and taking them apart, recognize and use slides and turns, describe spatial locations such as “above” and “behind,” and describe and use ideas of direction and distance in getting around in their environment (Clements, 1999a).

In the area of number, preschoolers can learn to count with understanding (Fuson, 1988; Gelman, 1994), recognize “how many” in small sets of objects (Clements, 1999b; Reich, Subrahmanyam, & Gelman, 1999), compare numbers (Griffin, Case, & Capodilupo, 1995), and learn simple ideas of addition and subtraction (Aubrey, 1997; Clements, 1984; Siegler, 1996). They can count higher and generally participate in a much more exciting and varied mathematics than usually considered (Ginsburg, Inoue, & Seo, 1999; Trafton & Hartman, 1997). Challenging number activities don’t just develop children’s number sense; they can

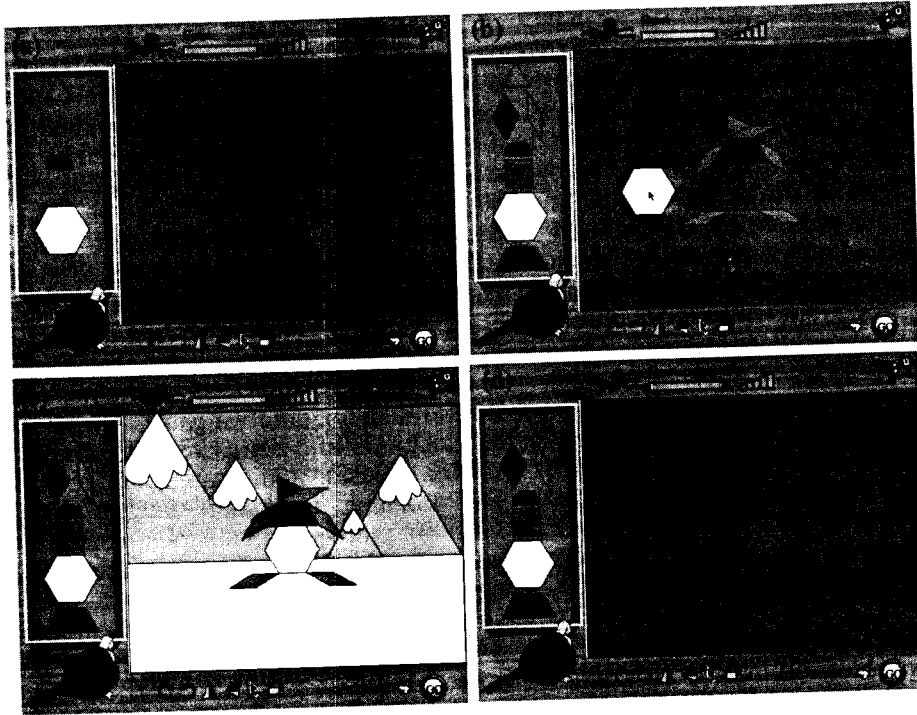


FIG. 15.2. Screens from the "Shape Puzzles" computed activity.

also develop children's competencies in such logical competencies as sorting and ordering (Clements, 1984).

As an illustration, *Building Blocks* includes a set of activities in which children learn one-to-one correspondence, counting, and equality. For example, children "get just enough" treats or scissors for the children at their table and in other real-world situations throughout the day. A computer activity challenges them to help a character get ready for a party, beginning with setting the table. At a higher level of the same activity, an on-screen character requests a certain number of items to add to the table. If a dish is missing, a character at the table may say, "I don't have a dish!" This type of natural feedback helps young children learn. In related activities, children decide how to share a given number of treats among those sitting at the table. Teachers might talk to the children about the way they are figuring out how many items they will need. Do they "deal out" the supplies each time? Or do they count the number of people, and use that number when getting supplies?

IN CLASSROOMS

We have completed the first three of the steps of classroom testing investigating components of the software design, assessing prototypes and curriculum (with one-on-one interviews with students and teachers), conducting pilot tests (in a few classrooms). In this section, I provide summaries and examples of the results of these tests.

The first pilot test for the geometry materials was limited in time—we had only 6 weeks (27 sessions) and so we attempted to condense the main *Building Blocks* activities for geometry. Pre- and posttests were given to the students. The children's average pretest score was 69. Their average posttest score was 84. Thus, despite limited exposure to the activities, there was a substantial average gain.

On what specific items did children make the greatest gains? They increased their scores on several difficult paper-and-pencil items in which they had to identify shapes, especially increasing in their ability not to be "fooled" by shapes that "looked like" triangles, squares, rectangles, and especially rhombi and quadrilaterals, but were not. In a similar vein, although they made only small gains in identifying figures that were congruent, they significantly increased their ability to correctly state that certain figures that looked to be congruent were actually not. On four items that asked children to make shapes with sticks, they increased moderately on initially choosing the correct number of sticks, while tripling their scores on making correct angles. Similarly, they substantially increased their ability to copy a complex design of three embedded shapes by manipulating transparent versions of the individual shapes.

What activities engendered this learning? The children in this group interacted with shapes in many different ways. They made pictures with paper cutouts of shapes, completed pattern block puzzles, searched for shapes in their environment and recorded what they saw (with adult help, if needed), sorted shapes, built shapes with straws and blocks, and identified shapes in storybooks. On computer they matched shapes, explored pattern block puzzles (including the use of geometric transformations), and identified shapes in the context of building Mystery Toys.

Let's look at a specific example. When sorting rectangles and nonrectangles, the teacher focused the children's attention on the sides. Chandra was able to tell the teacher which pile to put a new shape in, but was unable to articulate her sorting rule. Her partner, Marnie, similarly said, "It matches," while pointing to the correct pile. For the next shape (a right trapezoid), Chandra again pointed to the correct pile but gave color as her reason for doing so. Alethea joined the group and disagreed, saying the shape should go in the rectangle pile. Mitchell joined, pointed to the nonperpendicular side and said, "No, [it's not a rectangle, because] someone cut it right here."

Later, the teacher asked Tiffany if she knew the *name* of the trapezoid. She immediately said, "trapezoid," and pointed to the computer, indicating

where they had learned that vocabulary term. In the computer activity *Mystery Toys*, each shape name is pronounced as children match shapes. Later, the children are asked to click on the correct shape when the computer pronounces its name. This was a popular activity with the children and they enjoyed imitating the “computer” voice when they named shapes. Throughout the study, discussions encouraged children’s descriptions while encouraging the development of precise language. Early talk clarified the meanings of terms. With such clarification, children learned to explain why a shape belongs to a certain category—“It has three straight sides.” Eventually, they internalized such arguments, for example, thinking, “It is a weird, long triangle, but it has three straight sides!” Finding and identifying shapes by feeling was one particularly useful activity (see Fig. 15.3).

We piloted the number activities in another classroom of eight children; we altered the assessment extensively, so averages were not computed. Children made gains on all but a couple of items (most of which showed a “ceiling” or “floor” effect; e.g., counting verbally to 10) that remained the same from pre- to posttest. Increases were small (e.g., average gain of .25 points or more) on the following items:

- Recognizing the number of objects in collections of one to five (four children could recognize the number of small collections of one to five at pretest; the other four could recognize about half; all could recognize all collections on the posttest).



FIG. 15.3. “Feely box” activities support both physical exploration and the development of vocabulary and communication competencies.

- Counting 8 objects.
- Adding and subtracting small numbers (totals < 4) without objects.
- Ordering “towers” of 6 to 12 connecting cubes.
- Identifying the number between two numbers (e.g., “What number is between 5 and 7?”).

Children made moderate gains (average of 1/3 to 1/2 point) on the following items:

- Comparing the number in small collections (e.g., shown cards with ● ● ● and ● ● ● and asked, “Are these the same number?”).
- Recognizing counting mistakes that a doll made (moderate gains on two subitems; small gains on two subitems).
- Counting out (producing) sets of 6.
- Labeling sets of one to five with numerals.
- Recognizing which hidden set was larger when they were initially equal and items were added or subtracted.

Children made moderate gains (average above 1/2 point) on the following items:

- Quickly recognizing the number in small collections (i.e., subitizing; one to six objects).
- Adding and subtracting with objects (e.g., “If there were six dogs and only four bones, how many dogs wouldn’t get a bone?”).

What activities facilitated children’s learnings on the number items? Children counted objects continually throughout the study. Reading “non-math” books as well as books showing numerals were part of circle time, but books also became part of the centers. It was in the centers that children were able to interact with the books more extensively. Tanya was looking at the book *One Hungry Monster* and wanted to figure out how much food the monster ate on one of the pages. She put cubes near each numeral counting as she did, “1, ... 1,2 ... 1,2,3.” She then counted, “1,2,3,4,5,6 ... 6!” On computer, children’s counting was supported by the management system, which automatically adjusted the activity for difficulty and provided appropriate feedback and help.

The children had multiple opportunities to perform simple addition and subtraction. A toy dinosaur shop was set up in the socio-dramatic play area of the classroom. As Geri played with Janelle and Andre, she filled many “dinosaur orders.” This involved reading a numeral on a card and counting out the correct quantity for her “customers” and collecting the correct

amount of play money. Eventually, Janelle wanted to “trick” Geri and gave her two cards, a 2 and a 5. The teacher suggested Geri give Janelle two of one kind and five of another. She carefully counted out the two piles, put them together and counted the total. She then asked Janelle for \$7.

Ordering towers of connecting cubes became part of pretend play when children were “making stairs” for small characters to climb. On the computer, children moved along the learning trajectory built into the software, first “finding the next stair,” then building an entire staircase, then finding missing stairs.

Counting out sets of objects was another activity that the children had multiple opportunities to engage in throughout the year. In small groups, children placed the requested number of objects in play scenes, made cookies with chips, and filled dinosaur orders as previously described. In the beginning of the study, some children could only count out two objects. In these small groups, children worked on counting out different quantities depending on their capabilities. On computer, the children counted out chips on a cookie, silverware and plates for a party, or the correct number of dinosaurs to fill an order. The management system automatically adjusted the difficulty level (the number of items requested).

SUMMARY

Although mentioned briefly, it is easy to overlook the power of our combined strategies. Research-based computer tools stand at the base, providing computer analogs to critical mathematical ideas and processes. These are used, or implemented, with activities and a management system that guides children through fine-grained, research-based learning trajectories (developed over years of synthesizing our own and others’ empirical work). These activities-through-trajectories connect children’s informal knowledge to more formal school mathematics. The result is a package that will be motivating for children, but is also *comprehensive* in that it includes both exploratory environments that include specific tasks and guidance, building concepts and well-managed practice building skills, a full set of critical curriculum components, and a full range of mathematical activities. The initial pilot tests results indicate that such an approach can result in significant assessed learning gains consistent with the new *Standards* of the National Council of Teachers of Mathematics (a complete field test is being completed at the time of this writing).

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