Teaching Length Measurement: Research Challenges

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Many publications and curriculum materials promote a specific instructional sequence for introducing length measure: gross comparisons of length, measurement with nonstandard units such as paper clips, measurement with manipulative standard units, and finally measurement with standard instruments such as rulers. Several recent studies, however, imply that aspects of this traditional approach may not match children's developmental progression. This article describes several studies that challenge conventional wisdom regarding the teaching and learning of nonstandard and standard units, rulers, and measurement sense and draws educational implications from their results.

Third graders were making a map of their classroom. They wished to begin by measuring the room. Pleased, I passed out meter sticks. They began laying these down but soon stopped, puzzled.

"We need more."
"More meter sticks?" I inquired.
"Yeah. There's not enough."
"Maybe you could work together and solve that."
"No. Even all of 'em wouldn't reach."
"I mean, is there a way you could measure with just the meter sticks you have?"

After several minutes of futile attempts and useless hints, I believed I was miscommunicating. This led me to a demonstration.

"How about this? Can you lay a meter stick down, mark the end with your finger, and then move it?"
"Wow! Good idea!"

Their surprise and enthusiasm were delightful, but all the way home I thought, "How could this be new to them?" Familiar as I was with students' low performance and misconceptions in mathematics, this left a strong impression.

Students' Performance on Measurement Tasks

Many students use measurement instruments or count units in a rote fashion and apply formulas to attain answers without meaning (Clements & Battista, 1992). Less than 50% of seventh graders can determine the length of a line segment when the beginning of the ruler is not aligned at the beginning of the line segment. In international comparisons, U.S. students' performance in geometry and measurement is lower than in any other topic (National Center for Education Statistics, 1996).

What is the Problem?

My third grade map makers probably had few experiences measuring by iterating a unit. In how many homes are children making clothes or building with wood? School experiences are often also limited. Once, I encouraged the student teachers I was supervising to conduct measurement lessons. However, every cooperating teacher strongly suggested that the student teachers keep children at their desks and measure there, preferably on worksheets.

Are there additional reasons our students are weak in measurement? Many publications, including our own (Clements & Battista, 1986), have advised a specific instructional sequence: gross comparisons of length, measurement with nonstandard units such as paper clips, measurement with manipulative standard units, and finally measurement with standard instruments such as rulers. This approach is an educational tradition, based largely on Jean Piaget's theory of conservation—the idea that a physical quantity does not change during certain transformations. He and his collaborators found that children younger than 5 years judge length in terms of end points only (Piaget & Inhelder, 1967; Piaget, Inhelder, & Szeminska, 1960). For example, children judge a line segment and a bent path with the same end points to have the same length (Figure 1). If two equal length strips are cut as shown in Figure 2, younger children view one strip as longer. They may judge the left strip to be longer because it has longer pieces, or they may judge the right strip to be longer because it has more pieces (Carpenter & Lewis, 1976). Only later do children develop the ability to coordinate both the subdivision and reconnection of the parts and the ordering of the positions of the parts.
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Thus, the Piagetians claimed that children achieve an understanding of measurement only at about age 9.

Considering this alone, making sure children understand the logic of measurement before working with standard measuring devices such as rulers makes sense. Part of children’s charm, however, is their ability to surprise us. Do they actually need to develop Piagetian reasoning abilities of length conservation and transitivity before learning measurement? They do for some measurement concepts, but not for most (Hiebert, 1981; Hiebert, 1984; Petitto, 1990). Only certain tasks require general logical reasoning. One is knowing the inverse relationship between the size of the unit and the number of those units. Another is understanding the necessity of measuring with equal-length units.

If children do not possess such general logical reasoning, specific instruction helps little. However, for many tasks that appear to require general logical reasoning, children find their own strategy to measure correctly. These solution strategies do not necessarily match the structural logic of the task. For example, children use intermediate measurements to compare two lengths without explicitly asking the transitivity question. They move a unit to measure the length of an object and do not worry about whether the length is being conserved. Finally, children of all developmental levels solve simple measurement tasks that do not appear to rely heavily on general reasoning. So Piagetian reasoning abilities are not good measures of learning readiness for many measurement tasks; children can begin measuring without such abilities (Hiebert, 1981; Hiebert, 1984). Further, we must ask whether the traditional “logical” sequence is best.

Which First: Nonstandard or Standard Units and Tools?

Recent research questions whether the traditional nonstandard-then-standard-then-ruler sequence is wise. One group of researchers asked students in Years 1 to 3 (corresponding to U.S. Grades K-2) to explain their strategies on three tasks (Boulton-Lewis, Wilss, & Mutch, 1996). In Task 1 they gave students three pieces of string of lengths 27 cm, 27 cm, and 24 cm, along with a variety of standard and nonstandard measuring materials. They determined whether any of the pieces were the same length as another. Half the Year 1 students and most older students solved this task. In Task 2 researchers gave students a skip rope and asked them to find how long it was, so they could buy another “just like it.” This was difficult for Year 1 students but not older students. In Task 3 students were asked to compare two rows of matches. The standard row had five full-length matches glued in a straight line. This standard row (labeled C1 in Figure 3) was compared, in turn, to a straight row of 10 half-matches (C2), to a zigzag arrangement of full matches (C3), and finally to a zigzag arrangement of half-matches (C4). This was the most difficult task. The reasoning required on Task 3 is similar to that used with nonstandard units of measure. That is, students had to reconcile length and the number of units. The demand of the last two match tasks

...is similar to that which occurs when children are encouraged to use arbitrary units to measure so that they will understand the need for standard units. The strategies that children used for these tasks support the contention that this is not a good way initially to help children understand the need for standardized conventional units in the length measuring process. (Boulton-Lewis et al., p. 344-345)
Figure 3. Research tasks comparing the lengths of rows of matches.

For example, many children unsuccessfully used arbitrary devices such as placing a string around the shape C4 and cutting and comparing this length to the matches of C1. Most others used visual perception strategies, such as counting five matches in C1, then five in C4, saying that this came to halfway, and “proving” C4 had more by counting 1 to 10. Children were successful at an earlier age on the other tasks with or without standard devices. Therefore, using non-standard units early so that students understand the need for standardization may not be the best way to teach. If introduced early, children often use unproductive and misleading strategies that may interfere with their development of measurement concepts.

Just as interesting were students’ strategy preferences. Students of every age, especially in Years 1 and 3, preferred to use standard rulers, even through their teachers were encouraging them to use nonstandard units. One teacher did not allow use of rulers in her classroom, saying they had become a distraction because children wanted to use them!

Further, children measured correctly with a ruler before they could devise a measurement strategy using nonstandard units. To realize that arbitrary units are not reliable, a child must reconcile the varying lengths and numbers of arbitrary units. Emphasizing nonstandard units too early may defeat the purpose it is intended to achieve. That is, early emphasis on various nonstandard units may interfere with children’s development of basic measurement concepts required to understand the need for standard units. In contrast, using manipulative standard units, or even standard rulers, is less demanding and appears to be a more interesting and meaningful real-world activity for young children (Boulton-Lewis et al., 1996). These findings are not flukes—they have been supported by additional research (Boulton-Lewis, 1987; Clements, Battista, Sarama, Swaminathan, & McMillen, 1997; Clements, Sarama, & Battista, in press; Héraud, 1989).

Another study (Nunes, Light, & Mason, 1993) not only supports these findings, but also questions whether standard measurement instruments should be withheld until the end of the teaching sequence. The researchers investigated whether children could meaningfully use a ready-made system—rulers—before they “reinvent” such ideas as units and iteration. They had 6- to 8-year-old children communicate about measurement situations over a telephone. Each had a paper with a line segment drawn on it. They played a cooperative “game” in which their goal was to measure with an object to find out if the line on their sheet was longer, shorter, or equal to the line on their partner’s sheet. There were three situations, each containing a different object with which to measure. In each, the partners knew they had the same objects. For example, in the string situation, each had a string the same length as their partner’s. Across various tasks, the lines might be equal to the length of the string, double the length, or the like. The partners could use the string and discuss the task as much as they liked to determine if the lines on their
sheets were the same lengths. In the second situation, children had centimeter rulers. To determine whether children would use the ruler without understanding, in the third situation one child in each pair had a broken ruler starting at 4 cm, while the other had a normal ruler.

The traditional ruler supported the children's reasoning more effectively than the string; children's performance almost doubled. Their strategies and language (it is as long as the "little line [half] just after three") indicated that children gave "correct responses based on rigorous procedures, clearly profiting from the numerical representation available through the ruler" (p. 46). They even did better with the broken ruler than the string, showing that they were not just "reading numbers off" the ruler. The unusual situation confused children only 20% of the time. The researchers concluded that conventional units already chosen and built into the ruler do not make measurement more difficult. Indeed, children benefited from the numerical representation provided even by the broken ruler.

The Piagetian-based argument, that children must conserve length before they can make sense of ready-made systems such as rulers (or computer tools, such as those discussed in the following section), may be an overstatement. Findings of these studies support a Vygotskian perspective, in which rulers are viewed as cultural instruments children can appropriate. That is, children can use rulers, make them their own, and so build new mental tools. Not only do children prefer using rulers, but they can use them meaningfully and in combination with manipulable units to develop understanding of length measurement.

How Does Measurement Sense Develop?

Research also offers us a new look at a different topic: measurement sense. Measuring with tools is important, but so are other abilities, such as developing mental rulers. For example, many problems involve estimating or calculating the length of lines and drawing lines of a given length. For example, a teacher may ask students to draw figures with certain dimensions with the Logo turtle (Clements & Meredith, 1994). To command the turtle to draw an rectangle, they might give commands such as "fd 80" (draw a straight line 80 units in length in the direction you are headed) "rt 90" (turn right 90°), "fd 30 rt 90 fd 80 rt 90 fd 30 rt 90."

Third-grade students' strategies for solving such problems vary in sophistication (Clements et al., 1997).

Some students just guess, without making or marking any units. Others draw hash marks, dots, or line segments to partition lengths. That is, they create visible units they can count. For some children, however, the segments they mark off are not equal. The most sophisticated students do not mark off units. However, they are not just guessing. They draw proportional figures and visually partition line segments to assign them a length measure. They can visually segment distances and use part-whole strategies to find unknown lengths. They have an "internal" measurement tool. This is not a static image, but a mental process of moving along an object, segmenting it, and counting the segments. Students can impose such a "conceptual ruler" onto objects and geometric figures (Steffe, 1991). This is a critical point in their development of measurement sense.

Another way to develop measurement sense is to encourage students to forge connections between number and geometry. Logo's turtle graphics, for example, provide an arena in which young children may use the turtle step (a pixel) or centimeters as standard units (Campbell, 1987; Clements & Meredith, 1994). Walking and drawing paths, then commanding the turtle to draw paths may help students build units of length by segmenting continuous motion (Clements & Battista, 1992; Steffe, 1991). The emphasis on paths and movement might help students focus on intervals as units of length, instead of on points (Cannon, 1992) and so develop measurement concepts and measurement sense. So, as with rulers, Logo is another "ready-made system" children benefit from using.

As a specific example, a third grader, Anne, and her partner were observed playing "Coming Home" (Clements et al., 1997; see Figure 4). They had to instruct the turtle to visit objects on the screen and return by the same path using as few commands as possible. Anne said, "Try bk 80. Because we put fd 50 in [pointing at the Logo code], but then we had to put in a fd 30 [pointing first to the code, but immediately then to that section of the path], so it's got to be bk 80 to get back [gesturing back along the section of length 80]." Anne's actions showed clearly that she was connecting the geometric form of the turtle's path and the numeric representation of the Logo commands that created it. The Logo environments the girls were using, Turtle Math (Clements & Meredith, 1994), provided tools, such as the Label Lines tool, to assist her mathematical thinking. The dynamic links between geometry and number—any change in code automatically reflected in a corresponding change in the geometric figure—helped students construct connections between their own number and spatial ideas. When Anne tried her idea, the computer provided feedback, proving to her that her reasoning was right.
Implications for Teaching and Learning

The sequence in which children engage in measurement experiences (and we should conduct research to assess any new approaches) should be reconsidered. Even preschoolers can compare two objects directly and recognize equality or inequality of length (Boulton-Lewis et al., 1996). They should be given a variety of experiences comparing the size of objects; for example, finding all the objects in the room that are as long as their forearm.

After such experiences, children should measure, connecting number to length. Kindergarten and young primary-grade children prefer to use a standard measuring device even if they do not understand it fully or use it accurately. Teachers may consider allowing students to use rulers along with manipulable units such as centimeter cubes and arbitrary units. (The research does not indicate that using nonstandard units is harmful, just that using them with young children so they understand the need for standardization may be premature.) Accurate measuring procedures, such as placing manipulative units without leaving spaces between them, can be slowly developed. Similarly, with rulers, teachers can develop concepts and procedures such as accurate alignment (e.g., ignoring the gap at the beginning of many rulers), starting at zero, and focusing on the lengths of the units rather than only the numbers on the ruler. Counting points rather than line segments is more likely in ruler activities and partitioning tasks. The question, “What are you counting?” can not be overemphasized. That is, accepting earlier use of rulers is not the same as believing that such use implies mastery of either the tool or of measurement concepts (Lehrer, Jenkins, & Osana, in press). Rather, it is an additional way to present experiences and problems that will help children develop understanding. Using manipulable units to make their own rulers helps children connect their experiences and ideas.

Later, in second or third grade, teachers can introduce students explicitly to the ideas of the relationship between units and the need for standard units.
relationship between the size and number of units, the need for standardization of units, and additional measuring devices can be explored.

Primary grade children often make estimates on number lines based only on the number sequence, de-emphasizing the intervals between numbers. Of course, one can use a number line, as opposed to a ruler, without considering these intervals. Indeed, some educators have actually found that using such an "unlabelled number line," on which children write numerals useful to them, is superior in many situations (Gravemeijer, 1994). It is also helpful to be able to use a proportional number line. Only when children conserve length and learn the necessity of maintaining equal-length units can they develop and use such a sense of proportion in numerical values (Petitto, 1990). At this point, work with a proportional number line is most beneficial.

In a related vein, teachers should observe children's strategies for solving problems involving drawing and estimating lengths. Length tasks such as sketching a rectangle with particular dimensions may be presented, and teachers can observe whether students partition the lengths. Students who draw marks may need to have such perceptible units to quantify the length. These children can be presented with similar tasks, such as drawing a 10-by-5-cm rectangle, with an emphasis on equal-internal partitioning and the creation of different units of length.

Students who do not and cannot segment lines to iterate units and partition lengths can be guided to continually tie the results of that activity to their counting. For example, they might draw a toy, measure it, and draw it again using the same (and later, a smaller) measure. They could measure distances by counting their steps along a path. Teachers should emphasize experiences and ideas of motion and distance.

Finally, some students may be observed mentally partitioning lengths using a "conceptual ruler." They should be challenged with difficult tasks such as that in Figure 5. Children are to find all the "missing measures" and label the length of every line segment in the drawing. Logo turtle geometry experiences especially help students link number and geometry in measurement activities and build measurement sense. Figure 4 illustrates one turtle activity; problems such as that in Figure 5 also make good turtle tasks because children receive feedback on their reasoning. Turtle geometry provides both motivation and meaning for many length measurement activities. This illustrates an important general guideline: Students should use measurement as a means for achieving a goal, not only as an end in itself.

Conclusions

Measurement is one of the principal real-world applications of mathematics. It bridges two critical realms of mathematics: geometry or spatial relations and real numbers. Done well, education in measurement can connect these two realms, each providing conceptual support to the other. Indications are, however, that this potential is usually not realized. U.S. students study geometric measurement less than those in most other countries (National Center for Education Statistics, 1996).

Mathematics curriculum should include more geometric measurement, from building with wood to computer activities. Research-based activities should be designed to challenge traditional sequences of instruction. Teachers' action research can help us learn what these implications mean for different classrooms.

References


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Figure 5. Problem in which students must fill in all the missing measures (Clements et al., 1995).
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