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Young Children's Concepts of Shape

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We investigated criteria preschool children use to distinguish members of a class of shapes from other figures. We conducted individual clinical interviews of 97 children ages 3 to 6, emphasizing identification and descriptions of shapes and reasons for these identifications. We found that young children initially form schemas on the basis of feature analysis of visual forms. While these schemas are developing, children continue to rely primarily on visual matching to distinguish shapes. They are, however, also capable of recognizing components and simple properties of familiar shapes. Thus, evidence supports previous claims (Clements & Battista, 1992b) that a prerecognitive level exists before van Hiele Level 1 ("visual level") and that Level 1 should be reconceptualized as syncretic (i.e., a synthesis of verbal declarative and imagistic knowledge, each interacting with the other) instead of visual (Clements, 1992).

Key Words: Cognitive development; Conceptual knowledge; Constructivism; Content knowledge; Early childhood; Geometry; Learning theories

Extensive evaluations of mathematics learning indicate that elementary students are failing to learn basic geometric concepts and geometric problem solving, especially when compared to students from other nations (Kouba et al., 1988; Stigler, Lee, & Stevenson, 1990). Apparently, much learning of geometric concepts by U.S. students has been by rote; they frequently do not recognize components, properties, and relationships between properties (Clements & Battista, 1992b). One tenet of teaching for understanding is that one should build on a child's existing ideas. We investigated criteria preschool children use to distinguish members of a class of shapes from other figures.

Previous lines of inquiry concerning children's geometric conceptions have provided useful foundations but have also left gaps that impede the development of curriculum and the improvement of teaching. Three dominant lines of inquiry have been based on the theories of Piaget, the van Hieles, and cognitive psychologists (Clements & Battista, 1992b). One main theme of Piaget and

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Inhelder's (1967) influential theory on children's conceptions of space is that representations of space are constructed through the progressive organization of the child's motor and internalized actions. Therefore, one's representation of space is not a perceptual "reading off" of the spatial environment but is built up from prior active manipulation of that environment. This position generally has been supported by subsequent research (Clements & Battista, 1992b), and it serves as one foundation for this study.

According to the van Hiele theory, students progress through levels of thought in geometry when aided by instruction (van Hiele, 1986). Thinking develops from an initial, Gestalt-like visual level through increasingly sophisticated levels: descriptive and analytic, abstract and relational, formal deductive, and mathematically rigorous. At the visual level, children identify shapes according to appearance, recognizing them as visual gestalts using visual prototypes, saying, for instance, that a given figure is a rectangle because "it looks like a door." Children at this level do not attend to geometric properties or to traits that are characteristic of the class of figures represented. Clements and Battista (1992b) suggested a level, which they called *prerecognitive*, before the visual level. At this level, children may attend to only a subset of a shape's visual characteristics and are unable to identify many common shapes or distinguish among figures in the same class. Helping children move through these levels may be taken as a critical educational goal.

Cognitive psychologists have provided information about the classes children form, especially intuitively (Anderson, 1985). Evidence favors feature analysis (recognition of combinations of essential features) as opposed to template matching for recognition of perceptual patterns such as letters or shapes (Anderson, 1985; Gibson, Gibson, Pick, & Osser, 1962). Such feature analysis is unconscious and distributed (McClelland, Rumelhart, & the PDP Research Group, 1986), even if children also possess declarative knowledge (i.e., language-based knowledge about facts and things) (Anderson, 1983; Clements & Battista, 1992b) about the perceptual objects.

The knowledge base, however, is incomplete. Piagetian studies have not been grounded in educational concerns. Also, in much of Piagetian research the second main theme of this work, the *topological primacy hypothesis*, was investigated. This hypothesis—that geometric ideas develop from topological relations (e.g., connectedness, enclosure, and continuity) to projective (rectilinearity) and Euclidean (angularity, parallelism, and distance) relations—has not received strong support (Clements & Battista, 1992b; Geeslin & Shar, 1979; Martin, 1976). Piagetian interpretations of students' performance on tasks too often emphasize failures to coordinate multiple task demands at the expense of uncovering the development of ideas not yet differentiated and integrated; thus, new research is warranted. It appears that there are present at an early age certain Euclidean notions, for example, duplicating and recognizing Euclidean features (i.e., match and recognition), reconstructing such features from memory, and rotating and fusing shapes mentally (Rosser, Lane, & Mazzeo, 1988). Thus, contrary to the claims of Piaget and Inhelder and their interpreters (Peel, 1959), even preschool children should be able to work with such geometric ideas. Similarly, cognitive psychology has not been grounded in educational concerns. It has emphasized neither social-cultural factors nor the interrelationship of intuitive schemas and conceptual development.

In contrast, van Hielian research was grounded in educational concerns but did not deal with young children. In the original theory (van Hiele, 1986; van Hiele-Geldof, 1984) and in most of the subsequent research, the focus has been on students in middle school and beyond. Those who have studied young children support the notion that their geometric thinking is basically "visual," although this notion does not capture the variety of their responses (Clements, Battista, Sarama, & Swaminathan, 1997; Lehrer, Jenkins, & Osana, 1998). Indeed, Clements and Battista (1992b) postulated that a combination of the three perspectives is necessary, inasmuch as the van Hiele theory does not adequately describe young children's conceptions. Two important concerns are whether a level of geometric thinking (prerecognitive) exists before the visual level and what the nature of thought is at the early levels. Thus, research is needed to identify the specific, original intuitions and ideas that young children develop about geometric figures.

We designed this study to investigate the geometric concepts formed by young children. The goal was to answer the following questions: What criteria do preschool children use to distinguish members of a class of shapes (e.g., circles or triangles) from other figures? Do they use criteria in a consistent manner? Are the content, complexity, and stability of these criteria related to age or gender? What implications do findings have for theoretical descriptions of children's geometric thinking?

METHOD

Participants

Participants were 97 predominantly middle-class children, 48 boys and 49 girls, from two preschools and an elementary school with two kindergarten classes. The children were aged 3.5 (i.e., 3 years, 6 months) to 6.9 years and were divided into three groups according to age. Children younger than 4.5 years of age at the time of the study were grouped as the 4-year-olds ($n = 25$); children between 4.5 and 5.5 were grouped as the 5-year-olds ($n = 30$); and those above 5.5 were grouped as the 6-year-olds ($n = 42$).

Interview

Data were collected primarily through clinical interviews in a one-on-one setting during the first half of the spring semester. The focus of the interview was on the children's responses while they were performing shape-selection tasks. These were pencil-and-paper tasks in which the children were asked to "put a mark on each of the shapes that is a circle" on an $8\ ^1/_2 \times 11$ in. page of separate geometric figures (see Appendix). If the child was silent, the interviewer repeated the question. If the child did not react, the interviewer asked, “Do you know what a circle is?” All children replied affirmatively, and they were asked, “Can you find any on this page?” After the child marked some, the interviewer asked if there were any more. When that question was answered negatively, the interviewer asked questions such as the following: Why did you mark that one? How did you know that was a circle? I see you didn’t choose this one. Can you tell me why? A similar procedure was conducted for squares, triangles, and rectangles (most of the distractors were visually similar to the goal shape) and ending with circles and squares in a complex configuration of overlapping forms (included to provoke responses in a complex setting, not to assess ability to disembed per se). These tasks were selected from previous research on the van Hiele theory and the Agam program (a curriculum designed to develop the “visual language” of children ages 3 to 7 years) (Razel & Eylon, 1990). The format was maintained (e.g., pencil drawings instead of cutouts) to allow consistency with and comparison to previous studies using the same tasks and techniques[1] (Burger & Shaughnessy, 1986; Clements & Battista, 1992a; Razel & Eylon, 1991). Each interview, lasting about 20 minutes, was videotaped. The responses were scored and coded, and the data were analyzed to determine patterns and trends in the children’s understanding of geometric concepts.

The first data set was created by scoring children’s selections for correctness. The second data set resulted from an analysis of children’s verbalizations, both spontaneous and in response to the interviewer’s open-ended questions designed to clarify the criteria the children were using in making the selections (e.g., “How did you know that was [or was not] a rectangle?”). On the basis of the van Hiele theory and codes from previous research (Clements & Battista, 1992a; Lehrer, Osana, Jacobson, & Jenkins, 1993), children’s responses were coded into one of 22 response categories. Categories for responses that were not observed in this research (e.g., concerning properties dealing with angle size and relationships) were omitted from our tables. Conversely, if a particular response did not fit into a category, the researchers jointly either added a category or expanded an existing one. Each response category was classified in one of two superordinate categories, visual or property (see Table 1). A visual response was coded for any reference to a form’s looking like an object and for descriptions such as “pointy,” “round,” or “skinny.” A property response was one in which the child referred to the geometric components or properties of the form, such as “three sides” or “four sides the same length.” In cases of multiple responses about a single figure, the dominant response was coded when possible; if no response was dominant, we used “more than one response in the visual category” or “more than one response in the property category.” Early in the coding process, any doubt was resolved through the researchers’ group discussion. Multiple responses that spanned both categories were coded in the category that predominated; a simple

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[1] Our ongoing research is extending these inquiries to include manipulable shapes.

visual response immediately followed by one or more property responses was coded on the property level (consistent with the van Hiele theory). Later independent scoring and coding of the responses of two randomly chosen children resulted in 100% agreement by two of the authors. The verbal responses to each task were categorized into two groups, examples and nonexamples of the class (in the case of the nonexamples, children used the response to explain why the shape was not an exemplar), and two percentages were calculated for each: percentage of all responses made by that age group in the given category and percentage of children of that age group making the given response at least once.

Table 1
Coding of Verbal Responses of Children After Selection of Shape

Category	Verbal responses
Visual	Draws on paper or in air, saying, "It looks like this." "It looks [doesn't look] like a [shape name]." Reference to another shape on same page: "Same as this one." "Sort of like a [shape name]." Identifies another shape on page and declares, "This is not the same as that." Visual references to lines that are not horizontal or vertical: "It's slanty." "Pointy." or "It has corners." Looks like a [object name] Skinny/fat/long Big/small Orientation; gesturing along the diagonal of Shape 11 on the triangle task, saying, "It's too bent down." Miscellaneous visual: "It's a crazy one." More than one visual response
Property	States and indicates by gesture presence or absence of specific attribute or property (e.g., "It's a rectangle because it doesn't have a point on the top.") Round/curved/no straight sides/no corners (e.g., "It has a round top, so it's not a triangle.") Number of angles Number of sides Type of lines (e.g., "It has two lines going up and down.") Length of sides (e.g., "Two of the sides are the same length and the other two are the same length.") More than one property response
IDK	No response (NR) or "I don't know." "Just because."

Note. Adapted from Clements and Battista, 1992a, and Lehrer et al., 1993; codes not listed were not observed.

FINDINGS

Mean correctness scores for each shape-selection task are summarized in Table 2. Reliabilities (K-R 20) for this population are circles, .76; squares, .83; triangles, .55; rectangles, .82; and embedded circles and squares, .84. Correlations between scores across the tasks varied, ranging from $r = .15$ (circle and rectangle, the only correlation that was not statistically significant) to $r = .48$ (circle and square). Developmental and gender differences were assessed with analyses of variance for each task. There were no significant differences between boys and girls on the overall scores of any task.

Table 2
Mean Correctness Scores and Standard Deviations for the Five Shape-Selection Tasks

			Age groups		
Shapes	Possible score	Mean score (*SD*)	4-year-olds (*n* = 25)	5-year-olds (*n* = 30)	6-year-olds (*n* = 42)
Circles	15	14.41 (1.4)	13.76 (2.0)	14.33 (1.4)	14.86 (0.4)
Squares	13	11.30 (2.3)	10.64 (2.7)	11.17 (2.7)	11.79 (1.7)
Triangles	14	8.24 (2.5)	7.92 (2.7)	8.17 (2.6)	8.48 (2.2)
Rectangles	15	8.16 (3.2)	7.68 (3.9)	7.70 (2.9)	8.79 (2.9)
Circles and squares	28	17.24 (4.8)	13.68 (7.0)	17.40 (3.0)	19.24 (2.4)

The circle-selection task was the easiest for the children; their mean score was 14.41 out of a possible 15 (Table 2). A developmental difference was found in this task, with the 6-year-olds performing significantly better than the younger children ($F = 5.54$, $p < .005$).

Table 3 shows the percentages of children's verbal responses to examples and nonexamples of circles. Of all the responses 4-year-olds gave to examples of circles (see #1 in Appendix), 9% were coded as "Draws on paper or in air." Twenty percent of the 25 four-year-old children provided that response to at least one of the nine examples of circles. The ellipse (Shape 11) was the most distracting shape; 12% of the children marked it as a circle; almost all these children were either 4 or 5 years old. In addition, 20% of the 4-year-olds identified the curved shape (Shape 10) as a circle. Children, particularly the 4- and 5-year-olds, gave more verbal responses for the nonexamples than for examples of circles, possibly because it was easier for them to express differences from the strong imagistic prototype (Table 3). A large majority of children did not respond verbally to one or more items; thus the NR category constituted approximately one third to one half of all responses. For the examples of the circles, the dominant response category was "round, curved, no straight sides, no corners"; holistic visual justifications ("looks like") were most frequent for nonexemplars. There was no significant correlation between the number of children's visual or property-based responses and their correct selections; the "I don't know/no response" category (IDK) was significantly negatively correlated with selection accuracy ($r = -.45$, $p < .001$).

The mean score on the square-selection task was 11.3 out of a possible 13, indicating that the children were quite able to discriminate a square from the other forms on the page (Table 2). Although there was no significant overall developmental difference ($F = 1.98$, $p = .144$), 28% of the 4-year-olds and 13% of the 5-

Table 3
Percentage of Children's Verbal Responses to Examples and Nonexamples of Circles

	Examples of circles						Nonexamples of circles					
	4 years		5 years		6 years		4 years		5 years		6 years	
	(*n* = 25)		(*n* = 30)		(*n* = 42)		(*n* = 25)		(*n* = 30)		(*n* = 42)	
Verbal responses	*R*[a]	*C*[a]	*R*	*C*	*R*	*C*	*R*	*C*	*R*	*C*	*R*	*C*
Visual responses												
Draws	9	20	5	13	4	5	1	4	8	7	4	5
Looks like (shape)	1	4	3	7	11	10	10	40	17	50	17	14
Another shape on page	0	0	0	0	0	0	5	12	1	7	0	0
"Sort of" like	0	0	0	0	0	0	0	0	1	3	2	2
Not the same as	1	8	1	3	0	0	1	8	1	3	0	0
Slanty/diagonal/bent	0	0	0	0	0	0	1	4	1	3	0	0
Pointy corners	0	0	0	0	0	0	1	4	0	0	0	0
Looks like (object)	5	12	0	3	0	0	7	28	15	27	10	5
Skinny/fat/long	0	0	0	0	0	0	0	0	3	17	2	2
Reference to size	1	12	3	10	0	0	0	0	1	7	0	0
Orientation	0	0	0	0	0	0	0	0	0	0	0	0
Miscellaneous	1	4	1	7	0	0	1	8	2	10	4	5
Multiple	1	8	1	17	3	5	8	32	8	27	10	12
Property responses												
Presence attribute	0	0	1	3	3	2	6	20	4	13	4	5
Round/no sides	23	44	26	47	32	14	8	20	2	7	6	10
Number of corners	0	0	0	0	0	0	1	4	1	3	0	0
Number of sides	0	0	0	0	0	0	1	4	1	3	2	2
Type of line	0	0	0	0	0	0	0	0	0	0	0	0
Length of side	0	0	0	0	0	0	0	0	0	0	0	0
Multiple	0	0	0	0	0	0	1	8	1	4	0	0
NR/IDK/just because	59	84	51	90	47	90	49	80	34	60	38	98

[a] *R* is the percentage of all responses made by that age group in the given category. Column percentages do not all sum to 100 because of rounding errors. *C* is the percentage of children of that age group making the given response at least once. Column percentages do not all sum to 100 because children gave multiple responses.

year-olds, compared to only 5% of the 6-year-olds, identified the rhombus (Shape 3) as a square. In contrast, 63% of the 4- and 5-year-olds and 68% of the 6-year-olds accepted squares with no side horizontal (Shapes 5, 11, and 13) as squares. Again, a substantial number of verbal responses were categorized as IDK; the most frequent visual categories were "looks like" responses (especially frequent for the 4- and 5-year-olds) and, for nonexamples, multiple visual responses (Table 4). The property response "four sides" was proffered from 7% to 16% of the responses for examples (Table 4). The number of IDK responses was significantly negatively correlated to correct selections ($r = -.54$, $p < .001$); the number of visual responses was not significantly related. The number of property-based responses was significantly related to correct selection ($r = .34$, $p < .01$).

The triangle-selection task was more difficult for the children; the mean score was 8.24 out of a possible 14 (Table 2). There was no statistically significant difference in total correctness among the age groups ($F = 0.414$, $p = .662$). A developmental pattern among the three age groups and their selection of particular figures emerged. The 5-year-olds were more likely than either the 4- or 6-year-olds to correctly identify the examples of triangles (Shapes 1, 6, 8, 10, 11, 12) but also to

Table 4
Percentages of Children's Verbal Responses to Examples and Nonexamples of Squares

	Examples of squares						Nonexamples of squares					
	4 years		5 years		6 years		4 years		5 years		6 years	
	(n = 25)		(n = 30)		(n = 42)		(n = 25)		(n = 30)		(n = 42)	
Verbal responses	R^a	C^a	R	C	R	C	R	C	R	C	R	C
Visual responses												
Draws	6	16	5	13	0	0	2	8	5	13	0	0
Looks like (shape)	13	28	7	27	11	7	20	56	31	60	23	10
Another shape on page	4	12	6	17	6	5	5	20	2	7	0	0
"Sort of" like	0	0	0	3	0	0	1	4	1	3	0	0
Not the same as	2	8	0	3	0	0	1	4	1	3	0	0
Slanty/diagonal/bent	1	4	0	0	2	2	2	4	0	0	3	2
Pointy corners	0	0	0	3	0	0	1	4	1	3	3	2
Looks like (object)	2	8	0	0	2	21	4	20	3	12	10	5
Skinny/fat/long	0	0	0	0	0	0	1	4	1	3	0	0
Reference to size	0	0	3	10	0	0	0	0	2	3	0	0
Orientation	1	4	6	17	0	19	1	4	1	3	0	0
Miscellaneous	0	0	1	7	0	0	2	8	0	0	3	2
Multiple	1	4	4	20	3	5	6	28	11	30	20	12
Property responses												
Presence attribute	1	8	0	0	0	0	4	16	0	0	3	2
Round/no sides	0	0	0	0	0	0	1	4	0	0	0	0
Number of corners	3	12	7	10	6	2	3	8	2	10	0	0
Number of sides	7	20	15	17	16	7	2	12	3	10	0	0
Type of line	0	0	0	0	0	0	0	0	0	0	3	2
Length of side	0	0	1	7	2	2	1	4	1	3	3	2
Multiple	0	0	5	13	0	0	0	0	2	7	3	2
NR/IDK/just because	60	96	39	80	53	100	45	84	36	83	30	93

[a] *R* is the percentage of all responses made by that age group in the given category. *C* is the percentage of children of that age group making the given response at least once.

accept curved sides, either convex or concave (Shapes 3, 5, 7, 14). The 5-year-olds gave fewer responses in the IDK category, especially for nonexamples (Table 5). Visual responses constituted 34% to 47% of the responses, with no one visual category dominating. Property responses constituted 11% to 35% of the responses, with "number of corners and sides" dominating. Again, the children (particularly the 4- and 5-year-olds) provided more verbal responses for the nonexamples than for the examples of the triangles. The number of IDK responses was significantly negatively correlated to correct selections (r = -.39, p < .01); the sum of neither visual nor property responses was significantly related to correct selections.

The total mean score on the rectangle-selection task was 8.16 out of a possible 15 (Table 2); there was no significant difference between age groups (F = 1.405, p = .25). The 4-year-olds were more likely to accept the squares as rectangles. Shape 2 was selected by 28% of the 4-year-olds, compared to 17% and 10% of the 5- and 6-year-olds, respectively. Shape 7 was selected by 16% of the 4-year-olds, in contrast to 3% and 7% of the 5- and 6-year-olds, respectively. The children tended to accept "long" parallelograms or right trapezoids (Shapes 3, 6, 10,

Table 5

Percentages of Children's Verbal Responses to Examples and Nonexamples of Triangles

	Examples of triangles						Nonexamples of triangles					
	4 years		5 years		6 years		4 years		5 years		6 years	
	(*n* = 25)		(*n* = 30)		(*n* = 42)		(*n* = 25)		(*n* = 30)		(*n* = 42)	
Verbal responses	*R*[a]	*C*[a]	*R*	*C*	*R*	*C*	*R*	*C*	*R*	*C*	*R*	*C*
Visual responses												
Draws	7	20	8	23	0	0	9	28	7	23	3	2
Looks like (shape)	3	12	8	30	2	2	3	20	8	30	11	12
Another shape on page	3	16	3	10	2	2	9	40	9	37	5	7
"Sort of" like	1	4	1	3	0	0	2	12	3	17	2	2
Not the same as	1	4	0	0	0	0	1	4	0	0	0	0
Slanty/diagonal/bent	1	4	0	0	2	2	3	16	2	13	2	2
Pointy corners	3	8	0	0	6	7	1	4	0	0	0	0
Looks like (object)	1	4	1	3	2	2	2	12	2	13	8	10
Skinny/fat/long	0	0	2	10	2	2	1	4	2	13	0	0
Reference to size	3	16	1	3	6	7	1	4	0	0	0	0
Orientation	7	24	11	30	4	5	2	8	3	7	2	2
Miscellaneous	1	4	2	7	0	2	3	16	2	13	5	7
Multiple	3	16	10	33	15	7	7	28	7	33	9	10
Property responses												
Presence attribute	5	20	3	10	4	5	3	16	4	20	5	5
Round/no sides	0	0	0	0	0	2	1	4	0	3	3	2
Number of corners	5	16	5	10	4	2	4	16	13	40	2	2
Number of sides	3	12	11	27	13	10	2	12	10	27	14	7
Type of line	1	4	0	0	0	0	1	4	2	13	0	0
Length of side	0	0	0	0	0	5	0	0	0	0	0	0
Multiple	1	4	6	13	6	2	0	0	6	33	0	0
NR/IDK/just because	53	96	28	73	31	93	50	92	18	50	31	100

[a] *R* is the percentage of all responses made by that age group in the given category. *C* is the percentage of children of that age group making the given response at least once.

and 14) as rectangles; they were less likely to choose the shorter and other non-parallelogram forms as rectangles. Rectangles 9 and 12 were chosen by most children (76%, 76% of fours; 83%, 87% of fives; 93%, 95% of sixes). These data suggest that children have a prototype of a long rectangle, with only a weak bias toward perpendicularity. IDK responses dominated for 4-year-olds, and visual responses dominated for the 5- and 6-year-olds, confirming the children's apparent reliance on comparison to a visual image when distinguishing between forms (Table 6). Of the visual responses, children responded "It looks like [shape]" most frequently, although 6-year-olds frequently gave answers in the "skinny/fat/long" category for examples. Property responses ranged from 5% for 4-year-olds to 11% to 19% for 5- and 6-year-olds, with a greater ratio of the 5-year-olds proffering such a response at least once. Again, more verbal responses were given for nonexamples than for examples. The number of IDK responses was significantly negatively correlated to correct selections ($r = -.31$, $p < .05$); the number of visual responses was significantly related ($r = .32$, $p < .05$) and property responses were not significantly related to correct selection.

Table 6
Percentages of Children's Verbal Responses to Examples and Nonexamples of Rectangles

	Examples of rectangles						Nonexamples of rectangles					
	4 years		5 years		6 years		4 years		5 years		6 years	
	(*n* = 25)		(*n* = 30)		(*n* = 42)		(*n* = 25)		(*n* = 30)		(*n* = 42)	
Verbal responses	*R*[a]	*C*[a]	*R*	*C*	*R*	*C*	*R*	*C*	*R*	*C*	*R*	*C*
Visual responses												
Draws	2	8	4	13	3	2	5	36	5	30	3	5
Looks like (shape)	15	40	24	60	13	5	4	28	9	43	10	10
Another shape on page	2	0	2	7	0	0	3	16	7	33	2	2
"Sort of" like	0	0	2	3	6	5	1	12	0	3	1	2
Not the same as	1	4	1	3	0	0	0	4	0	3	0	0
Slanty/diagonal/bent	0	0	2	7	6	5	0	4	0	3	3	7
Pointy corners	1	4	1	3	3	2	1	12	1	7	1	2
Looks like (object)	0	0	2	7	0	0	3	24	6	27	8	7
Skinny/fat/long	1	4	2	7	19	7	1	8	1	7	6	5
Reference to size	1	4	2	7	0	0	0	0	1	7	0	0
Orientation	0	0	4	7	0	0	1	4	7	30	2	2
Miscellaneous	0	0	0	0	0	0	0	0	1	7	0	0
Multiple	3	12	5	10	6	5	3	20	5	33	11	10
Property responses												
Presence attribute	1	24	0	0	0	0	1	4	0	3	2	2
Round/no sides	0	0	0	0	0	0	0	0	0	0	1	2
Number of corners	2	4	1	3	0	0	3	4	2	17	0	0
Number of sides	0	0	7	13	6	5	1	8	4	20	2	2
Type of line	0	0	1	3	0	0	0	0	1	7	0	0
Length of side	0	0	4	10	6	5	0	0	1	13	5	2
Multiple	2	4	6	13	3	2	0	0	6	20	1	2
NR/IDK/just because	69	92	32	70	28	95	73	100	43	87	40	100

[a] *R* is the percentage of all responses made by that age group in the given category. *C* is the percentage of children of that age group making the given response at least once.

Finally, on the circle-square complex configuration, the mean score was 17.24 out of a total of 28 (Table 2). A significant difference was found among the age groups, with the 6-year-old children scoring better than those younger (F = 13.526, $p < .0001$). The younger children identified two of the ellipses, Shapes 6 and 14 (50% and 42%, respectively), as circles (recall that fewer children identified ellipses as circles in the circle-selection task). In this task, some shapes were embedded within other shapes; children were less likely to identify embedded circles and squares. For instance, Shape 8 (a square) has Shape 9 (another square) embedded within it, and Shape 8 is also divided into four quarters to create four more squares (Shapes 10, 11, 12, and 13). Although 32% of the children selected Shape 8, only 17% selected Shape 9, and even fewer selected Shapes 10, 11, 12, and 13. Few of the 4-year-olds, in particular, selected these embedded squares. The same was true with the circles. Whereas 76% of the children identified Shape 26 (an outer circle), only 17% identified the circle embedded within this shape (Shape 27). Also, only 35% of the children marked the square inside the circle (Shape 23), whereas 87% of the children selected the circle itself

(Shape 22). Overall, scores were lower on this embedded-figures task. Few children provided verbal responses for these shapes (2% to 11%), with few (0% to 6%) property-based responses (Table 7). A caveat is that some children showed signs of tiring on this task and may have been giving less attention to their selections. The number of IDK responses was significantly negatively correlated to correct selections ($r = -.68$, $p < .01$); the sum of neither visual nor property responses was significantly related to correct selections.

Table 7
Percentages of Children's Verbal Responses to Examples and Nonexamples of Circles and Squares

	Examples of circles/squares						Nonexamples					
	4 years		5 years		6 years		4 years		5 years		6 years	
	(*n* = 25)		(*n* = 30)		(*n* = 42)		(*n* = 25)		(*n* = 30)		(*n* = 42)	
Verbal responses	*R*[a]	*C*[a]	*R*	*C*	*R*	*C*	*R*	*C*	*R*	*C*	*R*	*C*
Visual responses												
Draws	9	4	5	3	0	0	2	8	0	0	0	0
Looks like (shape)	0	4	1	10	0	0	0	0	1	7	1	2
Another shape on page	0	0	1	3	0	0	0	0	1	3	0	0
"Sort of" like	0	0	0	0	0	0	0	0	0	0	1	2
Not the same as	0	0	0	0	0	0	0	0	0	0	0	0
Slanty/diagonal/bent	0	0	0	0	0	0	0	0	0	0	0	0
Pointy corners	0	0	2	3	0	0	0	0	0	0	0	0
Looks like (object)	1	4	0	0	0	0	1	8	1	3	0	0
Skinny/fat/long	0	0	1	3	0	0	0	0	1	3	0	0
Reference to size	0	4	0	3	0	0	0	0	1	3	0	0
Orientation	0	0	0	0	0	0	0	0	1	7	0	0
Miscellaneous	1	4	0	0	4	2	0	0	0	0	0	0
Multiple	0	0	0	0	0	0	0	0	1	3	0	0
Property responses												
Presence attribute	1	8	1	7	0	0	0	0	0	0	0	0
Round/no sides	1	4	5	10	0	0	0	0	2	10	0	0
Number of corners	0	0	0	0	0	0	0	0	0	0	0	0
Number of sides	0	0	0	0	0	0	0	0	0	0	0	0
Type of line	0	0	0	0	0	0	0	0	0	0	0	0
Length of side	0	0	0	0	0	0	0	0	0	0	0	0
Multiple	0	0	0	0	0	0	0	0	0	0	0	0
NR/IDK/just because	91	100	84	97	96	100	97	96	95	100	98	100

[a] *R* is the percentage of all responses made by that age group in the given category. *C* is the percentage of children of that age group making the given response at least once.

Across the tasks, then, the IDK category was consistently negatively correlated with selection accuracy, but the relationships of accuracy to visual and property responses varied with the task. Visual responses were significantly correlated only for rectangles; property responses were significantly correlated only for squares. Although not significant, two individual property categories, "presence of attribute" and "type of line," were more consistently correlated with accuracy across shapes.

A final statistical analysis was prompted by the desire to ascertain the consistency of children's verbal responses among the tasks. Correlations between the number of responses in each of the three broad categories were computed across tasks (Table 8). All 10 correlations between the total number of IDK responses were positive and significant; all 25 correlations between total IDK and visual responses were negative (only 9 significant); and 24 of 25 correlations between IDK and property responses were negative (but none significant). Likewise, 9 of 10 correlations between visual categories were positive (3 significant); all between-property categories were positive (4 significant). In contrast, visual to property correlations were mixed; 11 were positive (2 significant); 14 were negative (3 significant).

Table 8
Correlations Between Numbers of Verbal Responses

	D/c	V/c	P/c	D/r	V/r	P/r	D/s	V/s	P/s	D/t	V/t	P/t	D/cs	V/cs
V/c	-.25													
P/c	-.06	-.28*												
D/r	.50**	-.03	-.07											
V/r	-.28*	.10	.09	-.48**										
P/r	-.13	.21	.21	-.21	-.06									
D/s	.88**	-.16	-.15	.39**	-.24	-.09								
V/s	-.26*	.16	-.11	-.32*	.44**	-.21	-.18							
P/s	-.08	-.12	.52**	-.02	-.10	.45**	-.14	-.38**						
D/t	.71**	-.27*	-.01	.33*	-.01	-.15	.73**	-.03	-.15					
V/t	-.26*	.19	.02	-.21	.40**	-.26	-.22	.35**	-.22	-.34**				
P/t	-.17	.32*	.18	-.07	-.04	.69**	-.17	-.16	.49**	-.20	-.41**			
D/cs	.48**	-.31*	-.06	.44**	-.05	-.13	.54**	-.09	-.18	.77**	-.37**	-.19		
V/cs	-.12	-.12	.20	-.15	.10	.14	-.06	.23	-.09	-.12	.05	.01	-.11	
P/cs	-.07	.11	.10	.03	.02	.05	-.06	-.01	.02	-.07	.03	.05	-.08	.32*

Note. D/c indicates the I*D*K response (“I don’t know” or no response) for *c*ircles; other abbreviations are *V*, visual; *P*, property; *r*, rectangles; *s*, squares; *t*, triangles; *cs*, circles and squares.
$^{*}p < .05$. $^{**}p < .01$.

An additional pattern emerged across the shapes. On the square, triangle, and rectangle tasks, the children sometimes appeared not to distinguish the concepts of side and corner. A child would say that a form had four sides and then, when asked to count them, would count the corners. This practice was particularly prevalent among our youngest children and needs to be considered in further research.

DISCUSSION

We investigated criteria young children use to distinguish geometric shapes common in our social-cultural environment. Children identified circles with a high degree of accuracy. Six-year-olds performed significantly better than the younger children, who more frequently chose the ellipse and curved shape. Most children described circles as “round,” if they described them at all. Thus, the circle was easily recognized but difficult to describe for these children. Evidence indicates that they matched the shapes to a visual prototype.

Compared with their identification of circles, children's accuracy in identifying squares was only slightly less. Younger children were less accurate in classifying nonsquare rhombi but no less accurate in classifying squares without horizontal sides. Although only a minority of the children's reasons for selections referred to properties, there was a significant positive relationship between such responses and correct selections, suggesting that children are more likely to be accurate in their square identification when their reasoning is based on the shape's attributes.

Children were less accurate in recognizing triangles and rectangles than in recognizing circles and squares. One of the reasons to include the former tasks was for comparison to previous research. The comparable scores for 1,624 elementary students on the triangle task were 9.02, 9.03, 9.70, 10.07, 10.28, 11.47, and 11.34, for Grades K through 6, respectively (Clements & Battista, 1992a). The scores for students on the rectangle task were 9.47, 9.15, 9.12, 9.76, 9.67, 10.21, 10.18, for Grades K through 6, respectively (compare both with our scores in Table 2). Thus, across the two studies we observed a steady, but not large, increase from preschool to the intermediate grades.

Property responses were again present but infrequent, especially for rectangles. There was an inverse-U pattern in which 5-year-olds were more likely than younger or older children to accept both nonstandard triangles and those with curved sides. Children identified slightly more than half the rectangles correctly. The 4-year-olds were more likely to accept the squares as rectangles, possibly because they were less predisposed (because their prototype for rectangle was less distinguished from that for square) or less able to judge equality of all sides. Although the squares were included in the rectangle-recognition task (by the original task designers) to assess hierarchical inclusion, we did not expect or find such thinking in these young children. Their responses do show, however, that the path to such hierarchical thinking is a complex and twisting one with changes at several levels. This finding again raises the question of whether the strictly visual-prototype approach to teaching geometric shapes is a necessary prerequisite to more flexible categorical thinking or a detriment to the early development of such thinking. Kay (1987) provided first graders with instruction that (a) began with the more general case, quadrilaterals, proceeded to rectangles, and then to squares; (b) addressed the relevant characteristics of each class and the hierarchical relationships among classes; and (c) used terms embodying these relationships ("square-rectangle"). At the end of instruction, most students identified characteristics of quadrilaterals, rectangles, and squares and about half identified hierarchical relationships among these classes, although none had done so previously. Although the depth of these first-graders' understandings (especially of hierarchical relations) and the generalizations made on the basis of the empirical results must be questioned (Clements & Battista, 1992b), so too should we question the wisdom of the traditional, prototype-only approach, which may lay groundwork that must be overturned to develop hierarchical (abstract and relational van Hiele level) thinking.

All children tended to accept “long” quadrilaterals with at least one pair of parallel sides as rectangles. They referred to properties less frequently for rectangles than for triangles and squares; in addition, they were more likely to make accurate selections if they gave visual responses. The findings regarding squares and rectangles indicate a slow growth in the predisposition and ability to classify on the basis of right angles. These findings might illustrate the difficulty of the concept but might also result from the early introduction of squares and rectangles, and thus right angles, exclusive of attention to the angle concept, per se.

Children’s accuracy was lowest on the circle-square complex configuration. In addition, the 6-year-olds were significantly more accurate than the younger children (this result may reflect a development of field independence; the task was included to gather more information about children’s selections and descriptions in various situations). More children identified ellipses as circles in this complex and embedded configuration.

Although some developmental differences were found, they were found on only two of the five tasks, the circles and embedded tasks, and differentiated only the 6-year-olds from the 4- and 5-year-olds. Additional age differences were the greater rejection of nonsquare rhombi as squares by the 6-year-olds and greater acceptance of both triangles and nontriangles by 5-year-olds. Consistent with recent findings, our data showed no support for any hypothesis of gender difference in early geometric concept acquisition.

Correlational data have additional relevance. The high internal correlations for the tasks indicate their reliability and also a consistency in children’s responses that supports the theoretical position concerning levels of geometric thinking. Task scores were also correlated moderately with one another. Correlations of verbal responses similarly indicated moderate consistency, with IDK responses correlated positively with one another and negatively with visual responses. Visual and property responses were consistently positively related across tasks as well.

THEORETICAL AND EDUCATIONAL IMPLICATIONS

These results have two theoretical implications concerning children’s geometric understanding. First, the data support previous claims (Clements & Battista, 1992b) that a prerecognitive level exists before van Hiele Level 1 (visual level). (These findings also support the Piagetian position.) Children who cannot reliably distinguish circles, triangles, and squares from nonexemplars of those classes should be classified as prerecognitive; those who are learning to do so should be considered in *transition to,* instead of *at*, the visual level. We propose that children at this level are just starting to form schemas (networks of relationships connecting geometric concepts and processes in specific patterns) for the shapes. These early, unconscious schemas lead to pattern matching through feature analysis (Anderson, 1985; Gibson et al., 1962), even though the objects form undifferentiated, cohesive units in children’s experiences (cf. Smith, 1989). For example, nascent schemas may enable children to ascertain the presence of the features of closed and "rounded" to match circles, four near-equal sides with approximately right angles to match squares, and parallelism of opposite "long" sides to match rectangles. Later, other visual-spatial elements, such as the right angles of squares, are incorporated into these schemas and thus traditional prototypes may be produced. Further, older children can attend to these features separately, whereas younger children are not able or predisposed to focus on single features (Smith, 1989). Therefore, younger children can produce a prototype while identifying rectangles without attending to the components or specific features that constitute these prototypes. For children of all ages, the prototypes may be overgeneralized or undergeneralized compared to mathematical categorization, of course, depending on the exemplars and nonexemplars and the teaching acts children experience.

Second, the results support a reconceptualization of van Hiele Level 1. The high proportion of visual responses were in line with theoretical predictions. However, among these young children there is also evidence of recognition of components and properties of shapes, although these features may not be clearly defined (e.g., sides and corners). Some children appear to use both matching to a visual prototype (via feature analysis) and reasoning about components and properties to solve these selection tasks. Thus, through this study, we provide evidence that Level 1 geometric thinking as proposed by the van Hieles is more syncretic than visual, as Clements (1992) suggested. That is, this level is a synthesis of verbal declarative[2] and imagistic knowledge, each interacting with and enhancing the other. Thus, we suggest the term *syncretic level,* instead of *visual level,* signifying a global combination without analysis (e.g., analysis of the specific components and properties of figures). At the syncretic level, children more easily use declarative knowledge to explain why a particular figure is not a member of a class because the contrast between the figure and the visual prototype provokes descriptions of differences (Gibson, 1985). Children making the transition to the next level sometimes experienced conflict between the two parts of the combination (prototype matching vs. component and property analysis), leading to incorrect and inconsistent task performance.

In particular cases, when children made more references to attributes during the categorization tasks, they also made fewer correct selections. These errors were made because, instead of relying solely on comparison to a mental prototype, these older children began to rely on attributes that they had determined as defining the category. For example, many younger children called a figure a square because it "just looked like one," a typical holistic, visual response. However, some attended to relevant attributes; for them, a square had "four sides

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[2] We use *declarative knowledge* in the cognitive science sense of knowledge schemes of a propositional and language nature. One might have declarative knowledge of facts about shapes, and even about how to build shapes. In this study we did not preclude, but did not attempt to elicit, knowledge of making shapes.

the same and four points." Because they had not yet abstracted perpendicularity as an additional relevant and critical attribute, some accepted certain rhombi as squares. That is, even if their prototype has features of perpendicularity (or aspect ratio near 1), young children base judgments on similarity (i.e., near perpendicularity) instead of on identity (perpendicularity), and therefore they accept shapes that are "close enough" (Smith, 1989). The young child's neglect of such relevant (identity) attributes or reliance on irrelevant attributes leads to categorization errors. Mervis and Rosch (1981) theorized that generalizations based on similarity to highly representative exemplars will be the most accurate. This theory would account for the higher number of correct categorizations by those children who appeared to be making categorization decisions on the basis of comparison to a visual prototype without attention to irrelevant attributes. Finally, strong feature-based schemas and integrated declarative knowledge, along with other visual skills, may be necessary for high performance, especially in complex, embedded configurations. To form useful declarative knowledge, especially robust knowledge supporting transition to Level 2, children must construct and consciously attend to the components and properties of geometric shapes as cognitive objects (a learning process that requires mediation and is probably aided by using manipulatives for physical construction tasks).

Such a theory may explain the patterns of developmental differences in this study. We propose that children are developing stronger imagistic prototypes and gradually gaining verbal declarative knowledge. Those figures that are more symmetric and have fewer possible imagistic prototypes (circles and squares) are more amenable to the development of imagistic prototypes and thus show a straightforward improvement of identification accuracy. Rectangles and triangles have more possible prototypes. Rectangle identification may improve only over substantial periods of time. Similarly, recognition of shapes such as triangles, the least definable by imagistic prototypes of those we studied, may show complex patterns of development while the schema widens to accept more forms, over-widens, and then must be further constrained. Supporting this theory is evidence that the largest internal consistency was for those shapes with less visual and property variance within the class (circles and squares); the more variance (from rectangles to triangles), the less internal consistency we found.

Children's variegated responses (some visual, some property) may be another manifestation of this syncretic level. Further, they substantiate Clements's (1992) claim that geometric levels of thinking coexist. Progress through such levels is determined more by social influences, and specifically instruction, than by age-linked development. Although each higher level builds on the knowledge that constitutes lower levels, the nature of the levels does not preclude the instantiation and application of earlier levels in certain contexts (not necessarily limited to especially demanding or stressful contexts). For each level, there exists a probability for evocation for each of numerous sets of circumstances, but this process is codetermined by conscious metacognitive control, control that increases while one is moving up through the levels, so people have increasing choice to override the default probabilities. The use of different levels is environmentally adaptive; thus, the adjective *higher* should be understood as a higher level of abstraction and generality, without implying either inherent superiority or the abandonment of lower levels as a consequence of the development of higher levels of thinking. Nevertheless, the levels would represent veridical qualitative changes in behavior, especially the construction of mathematical representations (i.e., construction of geometric objects) from action.

Descriptions of children's early conceptions of geometric shapes are important not only for theory but also for teacher education (e.g., for cognitively guided instruction models) and for developers of constructivist-oriented curricula. Too often teachers and curriculum writers assume that students in early childhood classrooms have little or no knowledge of even simple shape identification (Thomas, 1982). Obviously, this belief is incorrect; preschool children exhibit working knowledge of simple geometric forms (even in the paper-based situations in the present study). Instruction should build on this knowledge and move beyond it. Students fail to reach the descriptive level of geometry in part because they are not offered geometric problems in their early years (van Hiele, 1987). The "prolonged period of geometric inactivity" (Wirszup, 1976, p. 85) of the early grades leads to "geometrically deprived" children (Fuys, Geddes, & Tischler, 1988).

Evidence supporting the existence of the hypothesized syncretic level and an earlier prerecognitive level is useful to researchers and to teachers of young children. The question should not be whether geometric thinking is visual or not visual but instead whether imagery is limited to unanalyzed, global visual patterns or includes flexible, dynamic, abstract, manipulable imagistic knowledge. Development of this latter type of knowledge and the concurrent development of active and reflective visualization (including the ability to consciously act on mental images, not just on drawings) is a viable goal at all levels of thinking, as is development of robust, explicit knowledge of the components and properties of geometric shapes, knowledge based on their existence as cognitive objects. We therefore recommend that teachers encourage students to describe why a figure belongs or does not belong to a shape category. Visual (prototype-based) descriptions should, of course, be expected and accepted, but property responses should also be encouraged. Such verbal descriptions may initially appear spontaneously for shapes with stronger and fewer prototypes (e.g., circle, square) and especially should be instructionally supported for those shape categories with more possible prototypes, such as triangles. In all cases, the traditional, prototype-only approach with limited exemplars should be rejected. Considering that in this study all age groups from 4-year-olds to 6-year-olds were equally accurate (about 2/3 correct) classifying squares without horizontal sides and that students perform poorly on similar items throughout the elementary grades (Clements & Battista, 1992a, 1992b), we suggest that the limited number of exemplars common in school materials impedes, and possibly undermines, students' continued development of rich schemas for certain geometric shapes.

Because of the limited verbalizations of these young children and the consequent ambiguity of the meaning of their utterances, these results are suggestive, not conclusive. We are presently conducting research using materials (e.g., manipulable shapes and construction tasks) and methodologies to address the remaining shortcomings in our knowledge of young children's geometric concepts of shapes.

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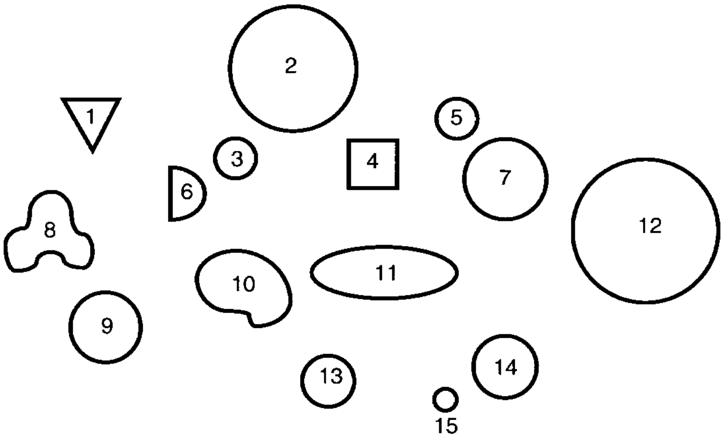
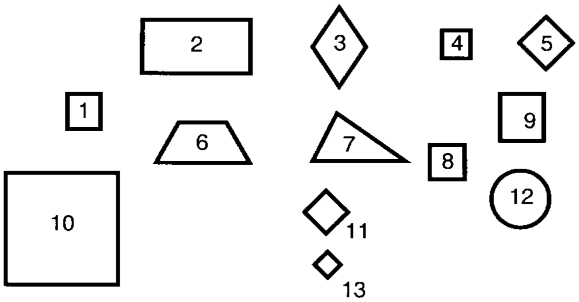
APPENDIX

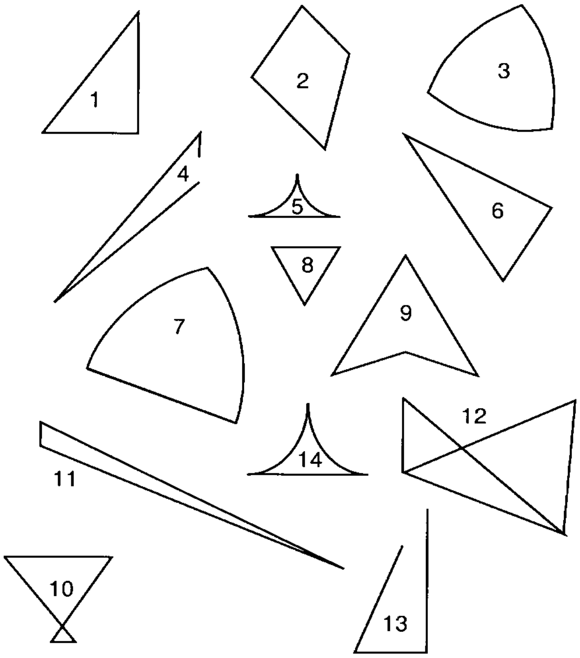
1. Student marks circles (Razel & Eylon, 1991).



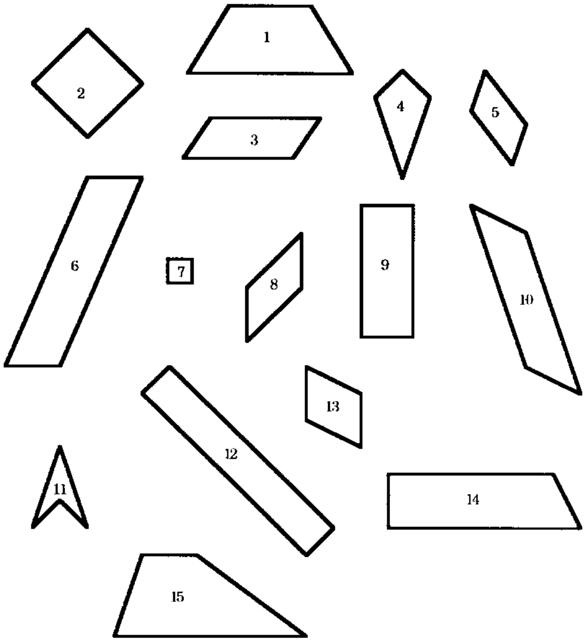
2. Student marks squares (Razel & Eylon, 1991).



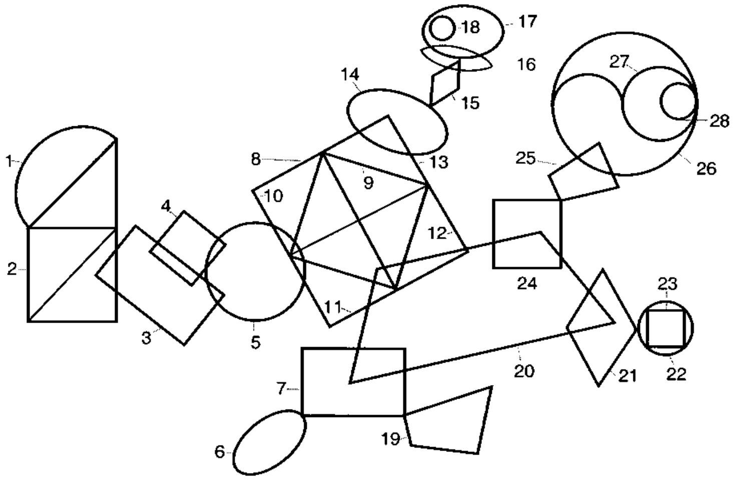
3. Student marks triangles (Burger & Shaughnessy, 1986; Clements & Battista, 1992a).



4. Student marks rectangles (Burger & Shaughnessy, 1986; Clements & Battista, 1992a).



5. Student marks first the circles, then the squares, with different colors (Razel & Eylon, 1991).



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