Forecasting the Salaries of Professional Personnel: An Application of Markov Analysis to School Finance

RAYMOND G. TAYLOR, JR. AND WILLIAM MICHAEL REID

In all organizations, preparation of the operating budget is a critical activity. Failure to estimate costs and revenues accurately leads to a variety of problems, from serious financial deficits to a general loss of confidence in the budgeting process. Personnel costs are likely to form the great majority of operating expenses within service organizations and, therefore, improved forecasts of personnel salaries are of significant benefit to school finance personnel.

This paper reports an improved methodology for estimating teacher salary costs well in advance of the fiscal year. It was tested in a community which had previously employed a more traditional method for budget development and was found to produce a superior forecast.

OVERVIEW OF EXPENDITURE FORECAST TECHNIQUES

Generally, local governments use one or more of the following four approaches when producing expenditure forecasts: expert judgment, trend extrapolations, deterministic techniques, and statistical methods. Within local government, school districts ordinarily employ deterministic techniques for forecasting teacher salary expenses. These methods generally involve the multiplication of the size of the teacher workforce by an anticipated average salary, perhaps broken down by various classes of teachers. These forecasts are usually then tempered by the judgments of experi-

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enced administrators regarding the likelihood of changes in the average salary or other significant factors such as an estimation of an inflation factor, an anticipated outcome of collective bargaining, a change in the size of the teacher workforce, or an unusually large number of expected retirements.

Statistical methods are rarely employed. When such methods are employed, the normal approach is to use standard regression and to accept the restrictive assumptions of that method, including linearity, homoscedasticity, and equal columnar variances. By contrast, the forecasting method used in this paper is based on the Markov technique, which although it has its own assumptions as given below, is relatively unrestricted.

Markov analysis, which has many applications in business and industrial decision-making, is one of several probabilistic models common to management science. However, the applications of Markov models in educational planning have been confined generally to: (1) student flows and enrollments, (2) planning for industrial education, (3) predicting the behavior of educational administrators, (4) forecasting teacher mobility within a large school system, (5) analyzing staffing policies, and (6) projecting housing values and racial mix for new school districts. There have been several treatments of Markov analysis in education planning. There have been no reports of applications of Markov analysis to the problem of forecasting personnel costs for budgetary use.


THE MARKOV MODEL

The use of the Markov technique requires several conditions, each of which was satisfied in the current study. First, there must be a finite number of discrete states. In this study, a teacher could be in any one of eight states: (1) newly employed, (2) in continuing employment, (3) resigned, (4) retired, (5) on leave, (6) on sabbatical, (7) ill for more than thirty days, or (8) deceased. Second, the transition probability of moving from one state to another in one time period (in this case, a year) must be independent of all prior states except the current one. Third, the total of all transition probabilities of moving from a given state to an alternate state in the next time period must equal 1.0. Fourth, these probabilities must be reasonably constant over time. Finally, transition matrices must be constructed. If they are to be generated by combining the historic behaviors of many individuals, as in the present study, then the individuals must belong to a group which is logically singular in relation to the behaviors being modeled. The teachers in this case were all employed using similar standards and met uniform school district criteria for continued employment, retirement, leaves, and sabbaticals. There is no reason to assume lack of homogeneity regarding illness and resignation. Death presumably is more likely for older teachers, but transition to this state turned out to be so improbable (one case in three years) that lack of homogeneity in this respect was disregarded.

STUDY METHODOLOGY AND FINDINGS

In this study, deterministic and probabilistic techniques were used to forecast personnel salaries. The deterministic technique compiled existing salary data, and then with the help of principals and supervisors, reviewed the status of each person in each position to project the likelihood of resignations, leaves, sabbaticals and other changes that would affect the budgeted amount needed to support each position. The probabilistic technique computed transition probabilities for each of the eight states to each of the remaining states and employed a Markov analysis as detailed below.

METHOD

For every teacher in the subject district (N=258), the state of the teacher was noted on October 1 for each of three successive academic years, 1983–86. Two transition matrices were computed, one for the first to second year and one for the second to third year. Very little variability was observed between the two, and so they were averaged into a single matrix for this study (Table 1).

At the end of the first year, the proportion of teachers in each state who were “active” (new, continuing, or ill) the year before was computed. Teachers were excluded who resigned, retired, were on sabbatical or leave, or who were deceased. The “active” proportions were treated, for computational purposes, as the beginning set of states (Table 2).

The Markov calculation begins by taking the proportion of individuals in the initial states (Table 2) and assigning them to the “from” categories at the left edge of Table 1. These proportions are then distributed across the rows of Table 1, multiplying each by the transition probabilities shown in the columns. The resulting products are then added down the columns to obtain a revised set of proportions. In a sense, the result is a new set of initial proportions adjusted for the effects of one time period. The new set of proportions represents the configuration of states one would expect the teachers to exhibit one year later if the initial probabilities were representative of all “beginning years.”

The Markov calculation is then continued by assigning the new proportions to the left edge of Table 1, redistributing and adding down. The redistribution process is repeated several times until the values before and after each set of calculations are nearly identical (within .00001, in this case). This final result is a set of “steady state” proportions, as given in Table 3. Here “steady state” means that the initial beginning effects have “worn off” and that the resulting probabilities, or proportions, are the most stable values available.

The final step in applying the Markov analysis technique to personnel forecasting is to count the number of teachers in each steady state category and to multiply such counts by their respective steady state proportions. The expected cost of each state should be known, or at least easily estimated. For example, the following definitions of expected costs were used for the present case study.

New. Average beginning salary was 1.10 times the base. In the year of illustration the base was $12,400. (1.10)(12,400) = 13,640
**TABLE 1**  
**TRANSITION PROBABILITY MATRIX**

<table>
<thead>
<tr>
<th>(FROM)</th>
<th>New</th>
<th>Continuing</th>
<th>Resigned</th>
<th>Retired</th>
<th>On leave</th>
<th>Sabbatical</th>
<th>Ill</th>
<th>Deceased</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>0.00000</td>
<td>0.77300</td>
<td>0.19400</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.03300</td>
<td>0.00000</td>
</tr>
<tr>
<td>Continuing</td>
<td>0.00000</td>
<td>0.89600</td>
<td>0.04000</td>
<td>0.01600</td>
<td>0.02200</td>
<td>0.00700</td>
<td>0.01700</td>
<td>0.00200</td>
</tr>
<tr>
<td>Resigned</td>
<td>1.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>Retired</td>
<td>1.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>On leave</td>
<td>0.00000</td>
<td>0.17800</td>
<td>0.53300</td>
<td>0.00000</td>
<td>0.28900</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>Sabbatical</td>
<td>0.00000</td>
<td>1.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>Ill</td>
<td>0.00000</td>
<td>0.80600</td>
<td>0.00000</td>
<td>0.13900</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.05500</td>
</tr>
<tr>
<td>Deceased</td>
<td>1.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>
### TABLE 2
**Initial Condition Probability Vector**

<table>
<thead>
<tr>
<th>STATES</th>
<th>New</th>
<th>Continuing</th>
<th>Resigned</th>
<th>Retired</th>
<th>On leave</th>
<th>Sabbatical</th>
<th>Ill</th>
<th>Deceased</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROPORTION</td>
<td>0.09300</td>
<td>0.75900</td>
<td>0.07800</td>
<td>0.01600</td>
<td>0.02300</td>
<td>0.00400</td>
<td>0.02300</td>
<td>0.00400</td>
</tr>
</tbody>
</table>

### TABLE 3
**Steady State Probability Vector**

<table>
<thead>
<tr>
<th>STATES</th>
<th>New</th>
<th>Continuing</th>
<th>Resigned</th>
<th>Retired</th>
<th>On leave</th>
<th>Sabbatical</th>
<th>Ill</th>
<th>Deceased</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROPORTION</td>
<td>0.07679</td>
<td>0.79904</td>
<td>0.06004</td>
<td>0.01516</td>
<td>0.02472</td>
<td>0.00559</td>
<td>0.01706</td>
<td>0.00160</td>
</tr>
</tbody>
</table>
Continuing. Average teacher salary was 1.60 times the base. $12,400 x 1.60 = $19,840

Resigned. 30 percent of the teachers who resigned were new and 70 percent were continuing. They were replaced by beginning teachers. Thus, the anticipated cost of resignations was $1.10(12,400) - .30(14,260) - .70(19,840) = - 4,526.

Retired. Retiring teachers were, on the average, near the top of the scale and earned about twice the base. Thus, the cost of retirements was the difference between a high-level salary and a beginning (replacement) salary. $1.1(12,400) - 2(12,400) = - 11,160

On leave. The cost of a leave was the difference between an average continuing salary and a beginning salary. $1.10(12,400) - 1.60(12,400) = - 6200

Sabbatical. The cost of a sabbatical was one half the teacher’s salary plus the cost of a beginning teacher. $(.5)(1.6)(12,400) + 1.1(12,400) = 23,560

Illness. Extended illness (thirty days or more) cost at least the teacher’s salary plus thirty substitute days. In the present illustration, the school system paid substitute teachers $40 per day, and discovered that new teachers are twice as likely as continuing teachers to be out more than thirty days for illness. Thus the cost was $(.67)(1.15)(12,400) + (.33)(1.60)(12,400) + 1200 = 17,301

Deceased. In the system used as an illustration, the probability of death among active teachers was based on a single incident and, therefore, a reliable estimate of cost could not be made. The cost of a retirement was used as a substitute which was then later multiplied by the steady state probability of deceased.

These expected costs for each state are then multiplied by the number of teachers in each state, and the results are totalled to give an overall projection (Table 4).

Case Study Findings

The deterministic and probabilistic models used to project teacher salaries were compared with actual experience to determine which made the best projection. The test year was 1985-86.

The 1985-86 budget projection for teacher salaries was $4,425 million using the deterministic, traditional, and commonsense method of increasing present salaries by factors such as anticipated collective bargaining agreements, probable retire-
ments, and early requests for leave. At the close of the 1985–86 school year, the actual expenditure for teacher salaries was $4.228 million. The Markov method forecasted a personnel salary budget of $4.308 million (Table 4).

**Conclusions**

The Markov approach provided a more accurate prediction of personnel costs for a school district than did the traditional deterministic technique for the one budget year studied. The authors believe that the application of this particular operations research method to forecasting local school budgets will result in more precise budgetary practices. Hence, it offers substantial benefits especially for districts that have to make projections well in advance of routine personnel decisions and cannot, therefore, confidently employ deterministic techniques. However, because the time period for this study was extremely circumscribed and because the use of Markov analysis in budget preparation is comparatively innovative, the authors encourage others to test this method more extensively and to report the results obtained before relying on it as a substitute for traditional methods.